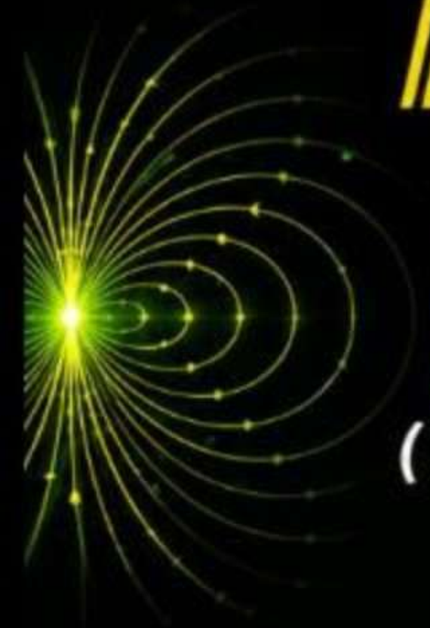
AKTU B.Tech Sem-3

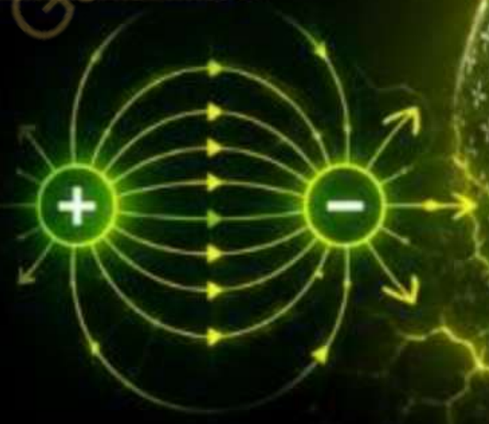
EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit-2: Lec-1





TOPICS TO BE COVERED

- Electric Field Intensity
- Electric Field due to a char ge distribution
- AKTU PYQs

GW
GATEWAY CLASSES

By Swati Ma'am

- M.Tech (Honours)
- B.Tech (Honours)
- GATE Qualified

Gateway Classes

Electrostatic Field

Electric Field :- Region surrounding an electric charge or group of charges.

Electrostatic Field → Electric field associated with stationary charges.

Two Laws Governing Electrostatics :-

Coulomb's Law

&

Gauss's Law

Gateway Classes

Coulomb's Law

Coulomb's Law states that the magnitude of the Electrostatic force between two point charges is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.

(Point charge - Size of charge is very small as compared to distance between them.)

Mathematical Form

$$F \propto q_1 q_2$$

$$\propto \frac{1}{r^2}$$

$$\propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

Gateway Classes

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GATEWAY CLASSES

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

where

ε_0 = Permittivity of free space

= 8.854×10^{-12} C^2/Nm^2

Numerically,

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ Nm^2/C^2$$

GatewayClasses

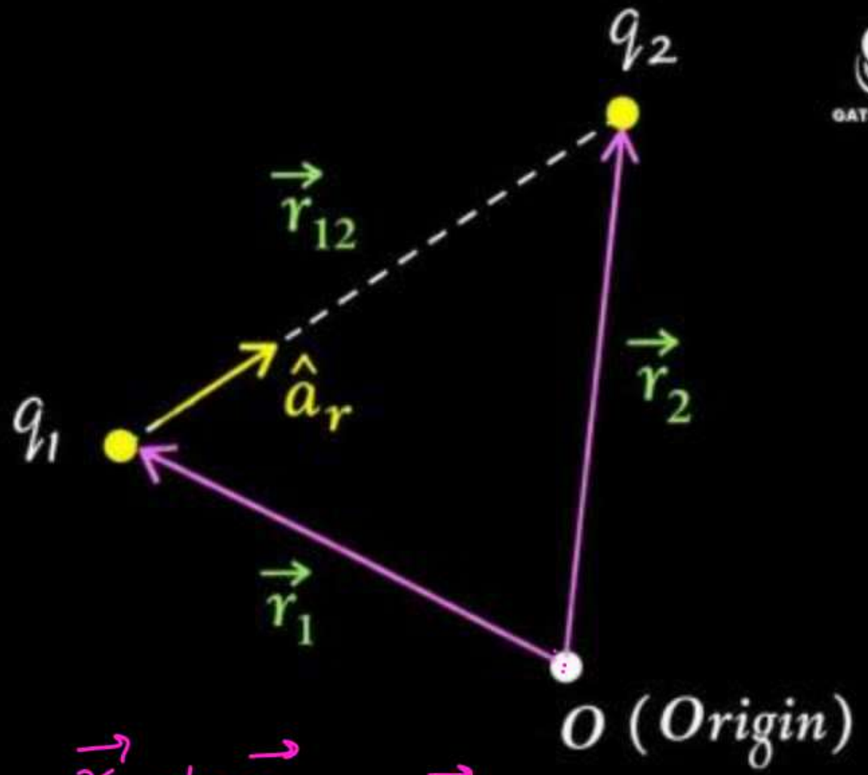
In Vector Form $\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{a}_r$

$$\hat{a}_r = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_{12}|^2} \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^2} \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$\boxed{\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}}$$



$\vec{r}_1 + \vec{r}_{12} = \vec{r}_2$

$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

Gateway Classes

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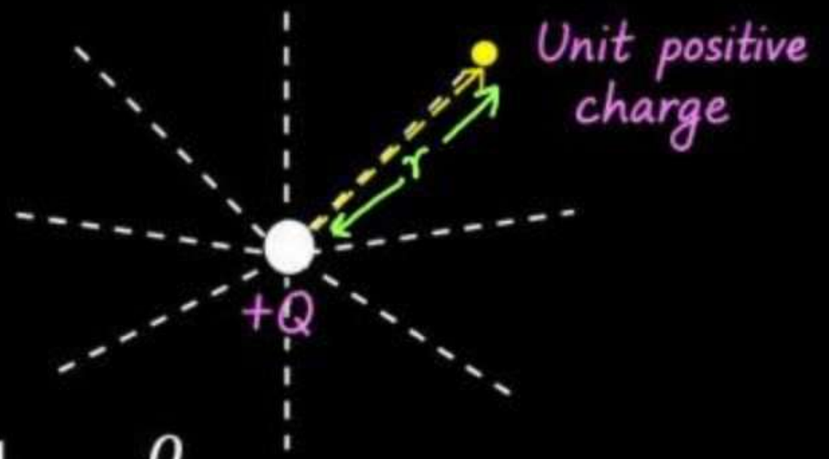
Electric Field Intensity

The intensity of electric field at a point in the field is defined as the force experienced by a unit positive charge placed at that point.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

q_0 → test charge



Unit :- Newton/Coulomb

q_0 :- Test charge

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \cdot q_0}{q_0 \cdot r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

Gateway Classes

For a point charge Q, if test charge is unit positive charge,

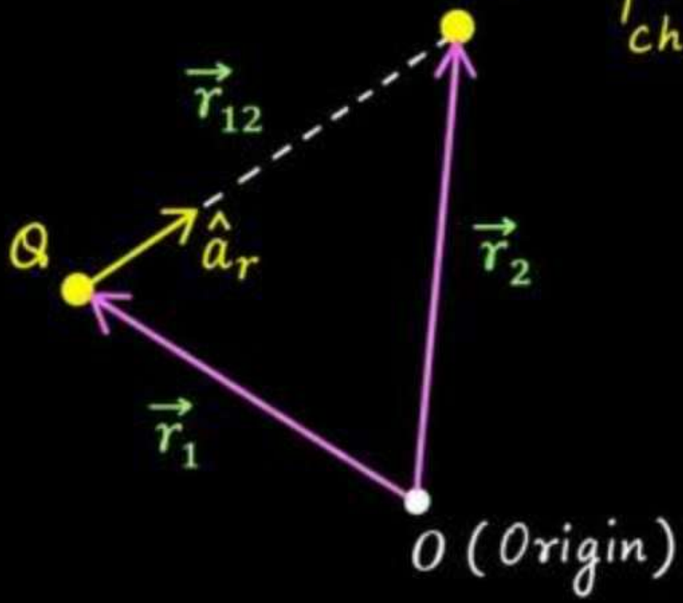
$$E = \frac{F}{1} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times 1}{r^2 \times 1} \Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}}$$

GW GATEWAY CLASSES

GW GATEWAY CLASSES

In Vector Form $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r_{12}^2} \hat{a}_r$

$$\hat{a}_r = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{|\vec{r}_{12}|^2} \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{|\vec{r}_2 - \vec{r}_1|^2} \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)}$$

Gateway Classes

Electric Fields due to Continuous charge distribution

1. Electric Field Intensity due to Infinite Length uniformly charged wire. (Infinitely Long Charged Wire)
2. Electric Field Intensity due to Semi-Infinite Line charge.
3. Electric Field Intensity due to Finite Line of Charge.
4. Electric Field Intensity due to Infinite Sheet of Charge.
5. Electric Field Intensity due to Uniformly Charged Sphere.

Gateway Classes

Electric Field Intensity due to Infinite Length uniformly charged wire

Derive an expression for Electric field intensity in space due to infinite length uniformly charged wire.

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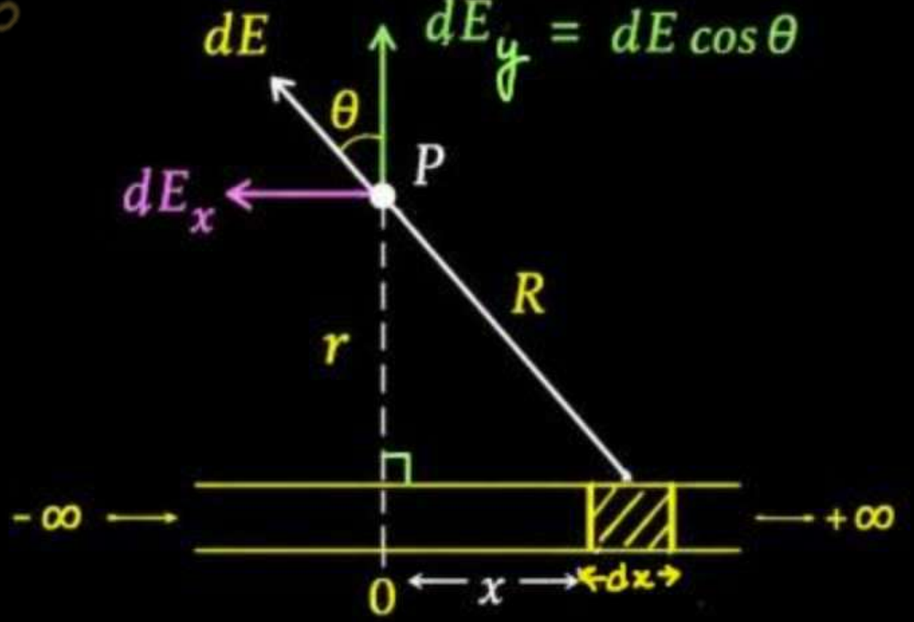
Consider an infinitely long horizontal wire along X-axis from -∞ to + ∞ ; Point P at distance r above wire on Y axis.

Electric Field due to small current element dx *at point P is*

$$dE = \frac{1}{4\pi\varepsilon_0} \times \frac{dq}{R^2} = \frac{1}{4\pi\varepsilon_0} \frac{\rho_L\, dx}{R^2} \quad -(1)$$

Now, $dE_x = -dE \sin\theta$ *-ve : -ve x axis*

$$dE_y = dE \cos\theta$$



Charge on this element

$$dq = \rho_L\, dx$$

Gateway Classes

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Due to symmetry, the horizontal components of the field intensity will cancel each other, only vertical components remain.

$\therefore$ wire is horizontal.

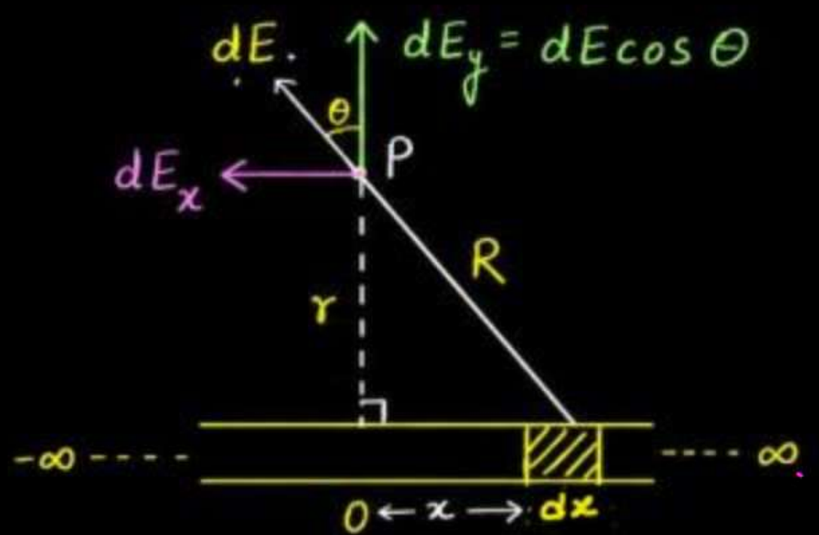
$\therefore$ E_p will be $dE\cos\theta$ in y direction

$$E_p \text{ or } E = \int_{-\infty}^{\infty} dE_y = \int_{-\infty}^{\infty} dE\cos\theta$$

$$= \int_{0}^{\infty} 2\, dE\cos\theta \quad -(2)$$

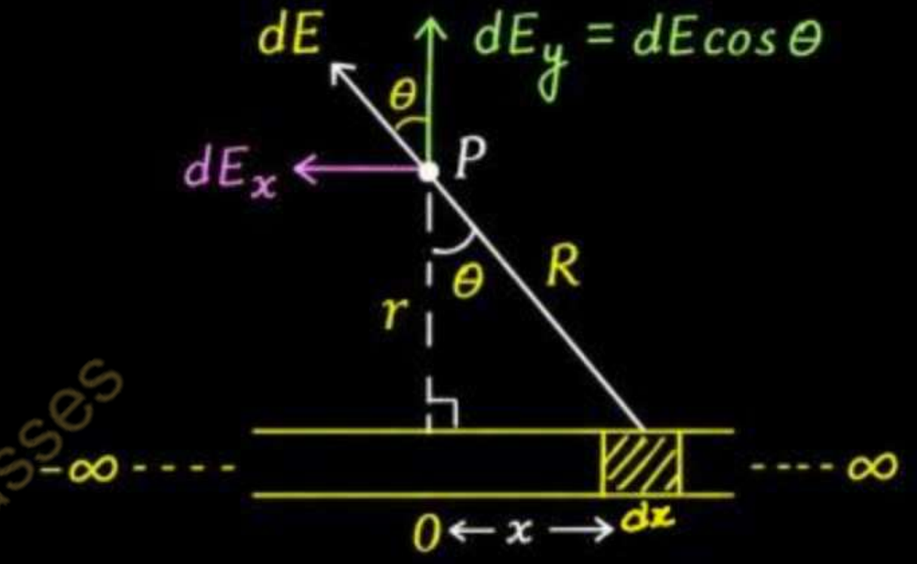
Substitute value of dE from (1) in (2)

$$= \int_{0}^{\infty} 2 \times \frac{1}{4\pi\epsilon_0} \frac{P_L \cdot dx}{R^2} \cdot \cos\theta$$



Gateway Classes

$$E = \int_0^{\infty} 2 \times \frac{1}{4\pi\epsilon_0} \frac{\rho_L \, dx}{R^2} \cdot \cos\theta \quad -(3)$$



From fig.

$$\tan\theta = \frac{x}{r}$$

$$x = r\tan\theta$$

$$dx = r\sec^2\theta \, d\theta \quad -(4)$$

$$R^2 = r^2 + x^2$$

$$= r^2 + r^2\tan^2\theta$$

$$= r^2(1 + \tan^2\theta)$$

$$R^2 = r^2\sec^2\theta \quad -(5)$$

→ Changing limits

when $x = 0$, $\longrightarrow \theta = 0$

when $x = \infty$, $\longrightarrow \theta = \pi/2$

$\therefore x = r\tan\theta$

Gateway Classes

Put ④ & ⑤ in ③, & change limits.

$$E = \int_0^{\pi/2} 2 \times \frac{1}{4\pi\epsilon_0} \cdot \frac{\rho_L \cdot r \sec^2\theta \cdot d\theta}{r^2 \sec^2\theta} \cdot \cos\theta$$

$$= \frac{\rho_L}{2\pi\epsilon_0 r} \int_0^{\pi/2} \cos\theta \, d\theta = \frac{\rho_L}{2\pi\epsilon_0 r} \left[\sin\theta \right]_0^{\pi/2}$$

$$= \frac{\rho_L}{2\pi\epsilon_0 r} \left[\sin\frac{\pi}{2} - \sin 0 \right] = \frac{\rho_L}{2\pi\epsilon_0 r} \left[1 - 0 \right] \Rightarrow \boxed{E = \frac{\rho_L}{2\pi\epsilon_0 r}}$$

Gateway Classes

In Vector Form

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r} \cdot \hat{a}_r$$

Direction

The direction of Electric field is outward if the wire carries a positive charge , and inward If it carries a negative charge.

Gateway Classes

Electric Field Intensity due to Semi-Infinite Line charge.

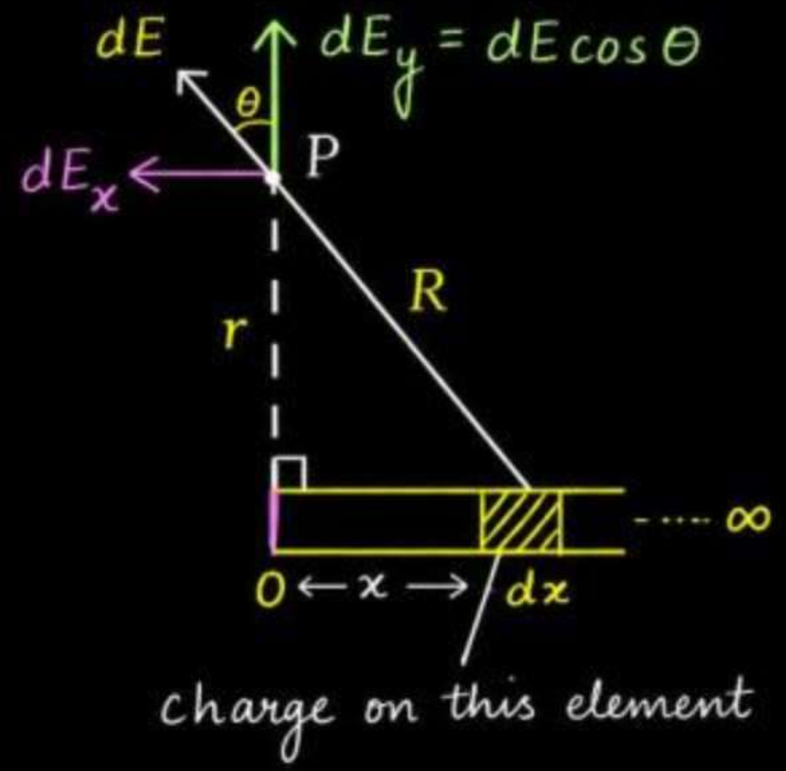
Consider a semi-infinite long horizontal wire . ↘ 0 to ∞

Electric Field due to small current element dx at point P is

$$dE = \frac{1}{4\pi\varepsilon_o} \times \frac{dq}{R^2} = \frac{1}{4\pi\varepsilon_o} \cdot \frac{\rho_L \cdot dx}{R^2} \quad -(1)$$

Now, $dE_x = -\,dE \sin\theta$ $\quad$ −ve : −ve x axis

$$dE_y = dE \cos\theta$$



Gateway Classes

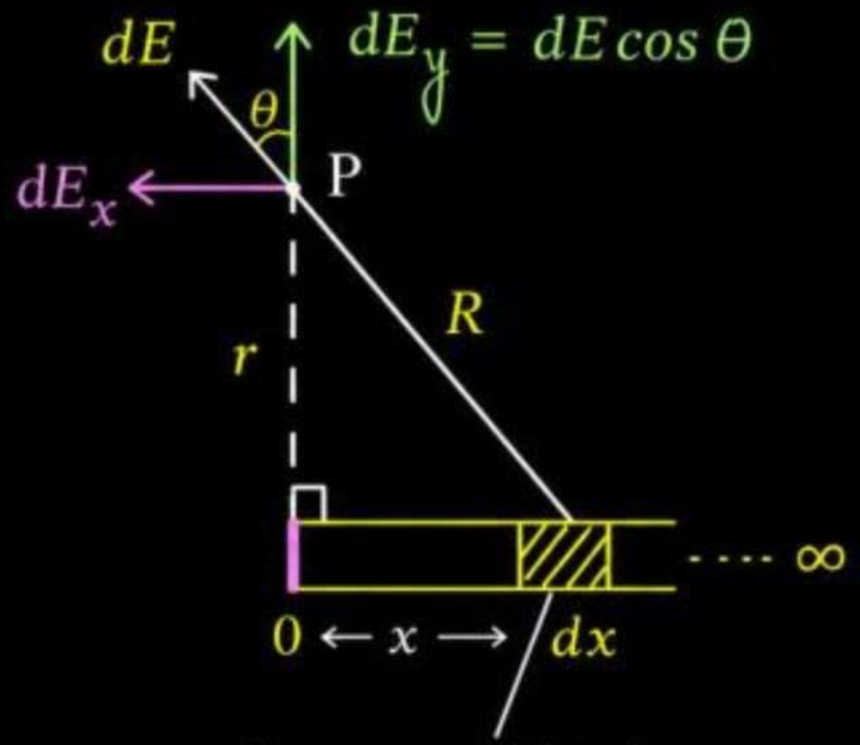
$\therefore$ E_p will be $dE\cos\theta$ in y direction

$$E_p \text{ or } E = \int_0^\infty dE_y = \int_0^\infty dE\cos\theta$$

$$= \int_0^\infty dE\cos\theta \quad -(2)$$

Substitute value of dE from (1) in (2)

$$= \int_0^\infty \frac{1}{4\pi\varepsilon_0}\frac{\rho_L\, dx}{R^2}\cdot\cos\theta$$

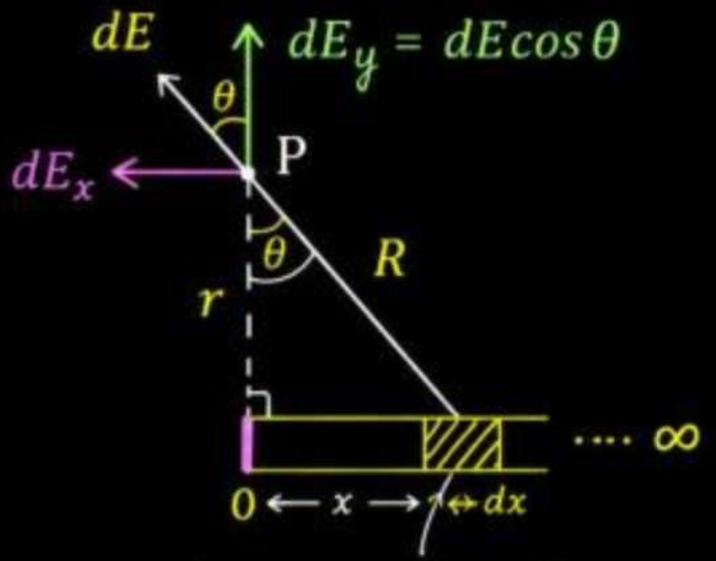


charge on this element

$$dq = \rho_L\, dx$$

Gateway Classes

$$E = \int_0^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{\rho_L \, dx}{R^2} \cos\theta \quad -(3)$$



charge on this element

$dq = \rho_L \, dx$

From fig.

$$\tan\theta = \frac{x}{r}$$

$$x = r\tan\theta$$

$$dx = r\sec^2\theta \, d\theta \quad -(4)$$

$$R^2 = r^2 + x^2$$

$$= r^2 + r^2\tan^2\theta$$

$$= r^2(1 + \tan^2\theta)$$

$$R^2 = r^2\sec^2\theta \quad -(5)$$

⟶ Changing limits

when $x = 0$, ⟶ $\theta = 0$

when $x = \infty$, ⟶ $\theta = \pi/2$

$\therefore \; x = r\tan\theta$

Gateway Classes

Put ④ & ⑤ in ③, & change limits.

$$E = \int_0^{\pi/2} \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho_L \, r \sec^2\theta \, d\theta}{r^2 \sec^2\theta} \cdot \cos\theta$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \int_0^{\pi/2} \cos\theta \; d\theta = \frac{\rho_L}{4\pi\varepsilon_0 r} \Big[\sin\theta \Big]_0^{\pi/2}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \left[\sin\frac{\pi}{2} - \sin 0 \right] = \frac{\rho_L}{4\pi\varepsilon_0 r} \left[1 - 0 \right] \Rightarrow \boxed{E = \frac{\rho_L}{4\pi\varepsilon_0 r}}$$

In vector form, $\boxed{\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0 r} \hat{a}_r}$

Gateway Classes

Electric Field Intensity due to Finite Line of Charge

Consider a finite line of charge AB of length L , midpoint at origin O.

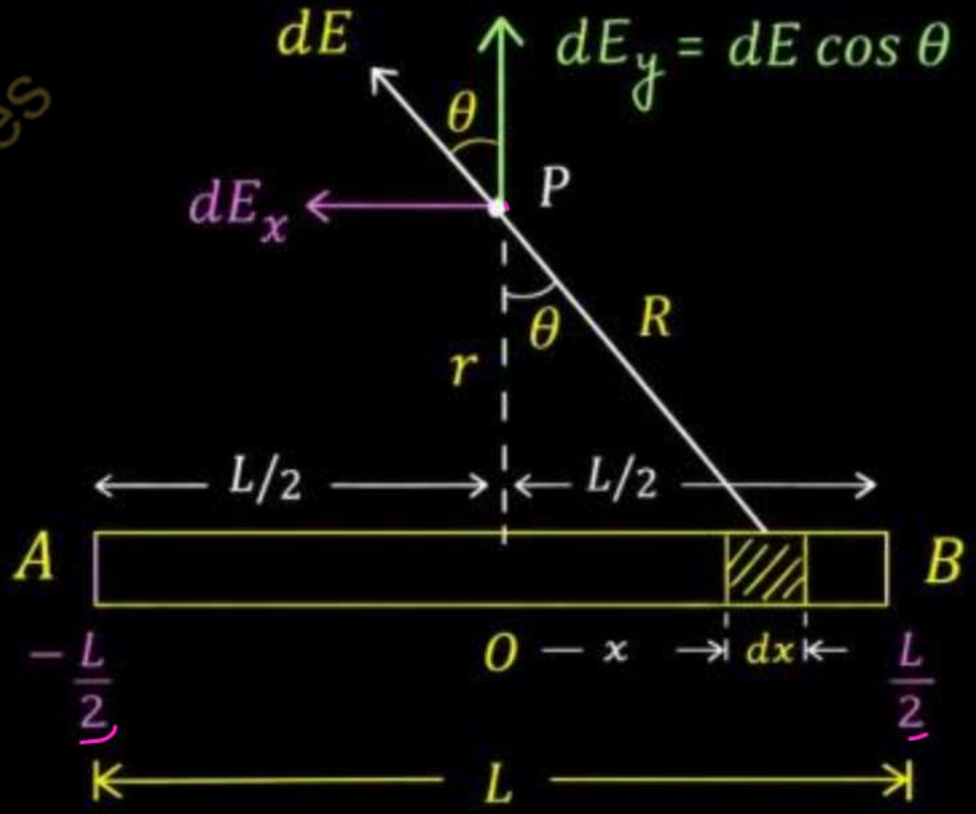
Point P at perpendicular distance r from the line.

Element dx at a distance x.

Components dE_x , dE_y with $dE_y = dE\cos\theta$

Limits $-L/2$ to $+L/2$

$$E = \int_{-L/2}^{L/2} dE_y = \int_{-L/2}^{L/2} dE\cos\theta$$



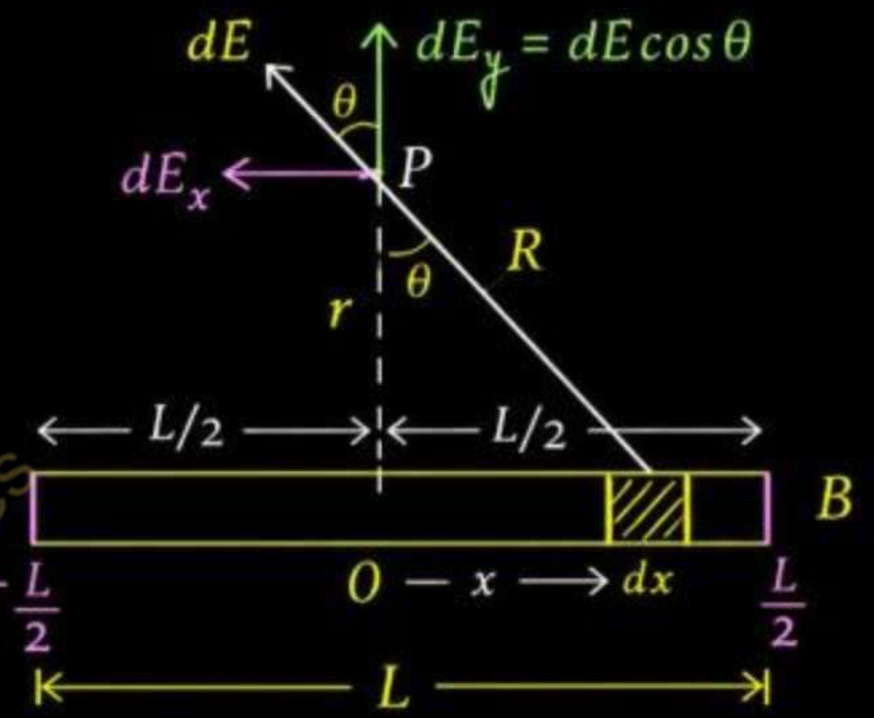
Gateway Classes

From (1)

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\rho_L \, dx}{R^2}$$

$$\therefore \quad E = \int_{-L/2}^{L/2} \frac{1}{4\pi\varepsilon_0} \frac{\rho_L \, dx}{R^2} \cos\theta$$

$$E = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{dx \cos\theta}{R^2}$$



From fig.

$$\cos\theta = \frac{r}{\sqrt{r^2 + x^2}} \quad \& \quad R^2 = (r^2 + x^2)$$

$$\therefore \quad E = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{r}{(r^2 + x^2)^{3/2}} \, dx$$

Gateway Classes

$$E = \frac{\rho_L}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{r}{(r^2+x^2)^{3/2}}\, dx$$

$$E = \frac{\rho_L}{4\pi\epsilon_0} \int_{0}^{L/2} \frac{2r}{(r^2+x^2)^{3/2}}\, dx$$

$$E = \frac{\rho_L\, r}{2\pi\epsilon_0} \int_{0}^{L/2} \frac{1}{(r^2+x^2)^{3/2}}\, dx$$

$$E = \frac{\rho_L\, r}{2\pi\epsilon_0} \left[\frac{1}{r^2} \cdot \frac{x}{(r^2+x^2)^{1/2}} \right]_0^{L/2}$$

Put $x = \frac{L}{2}$

$$E = \frac{\rho_L}{2\pi\epsilon_0 r} \cdot \frac{L}{(L^2+4r^2)^{1/2}}$$

Gateway Classes

Electric Field Intensity due to Infinite Sheet of Charge

Consider an infinitely long sheet of charge. It can be considered as a collection of multiple infinite line of charge.

Surface Charge density

$$\rho_S = \frac{charge}{area} = \frac{charge}{\ell \times w} \Rightarrow \rho_s = \frac{\rho_L}{w} \Rightarrow \rho_L = \rho_S \cdot w$$

$$\rho_L = \rho_S \, dy \quad \text{(on small strip)}$$

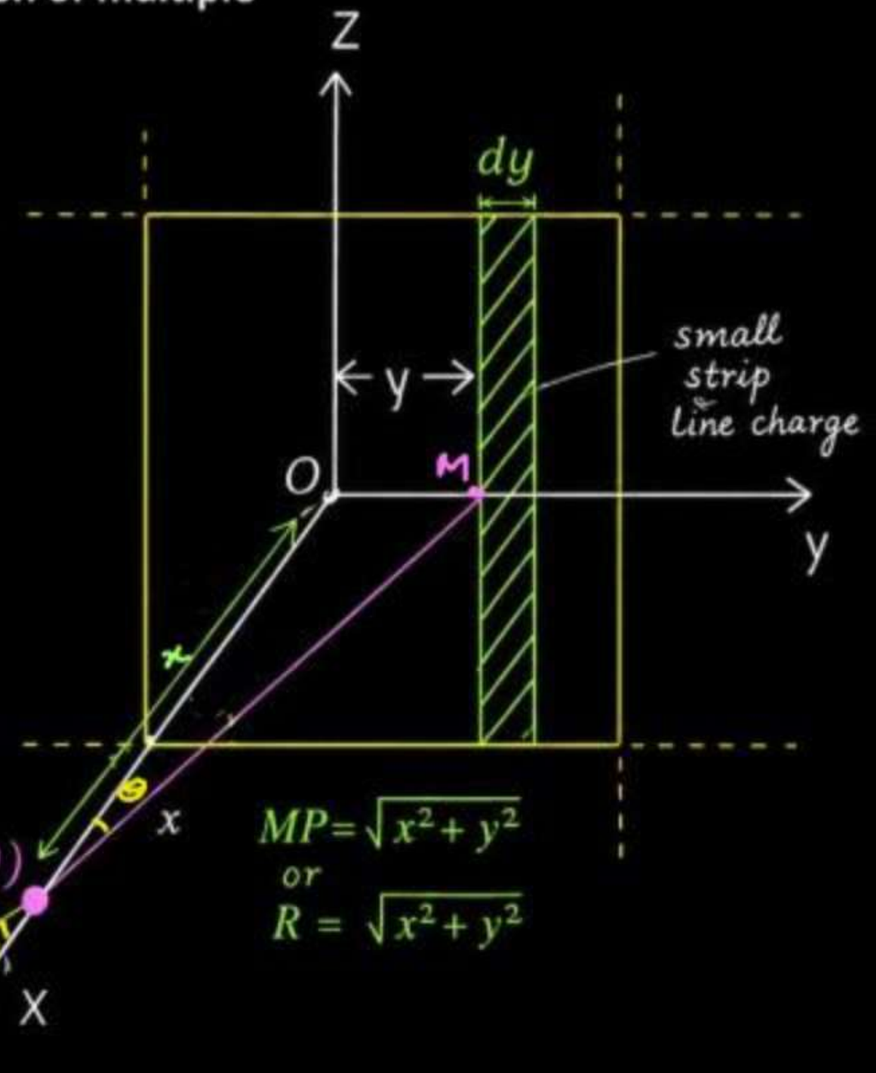
$$\rho_L = \rho_S \, dy \quad \text{(strip width } dy\text{)}$$

For a line charge,

$$E_x = \int_{-\infty}^{\infty} dE_x = \int_{-\infty}^{\infty} dE \cos\theta = \int_{-\infty}^{\infty} \left(\frac{\rho_s \, dy}{2\pi\varepsilon_0 \sqrt{x^2 + y^2}} \right) \cdot \cos\theta$$

dE (From (1) derivation)

Here MP (distance of strip from P) is taken → is on X axis



Gateway Classes

$$E_x = \int_{-\infty}^{\infty} \frac{\rho_s \, dy}{2\pi\epsilon_0\sqrt{x^2+y^2}} \cos\theta$$

Now, from fig.

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

$$E_x = \int_{-\infty}^{\infty} \frac{\rho_s}{2\pi\epsilon_0} \frac{x}{(x^2+y^2)} dy$$

$$E_x = \frac{\rho_s \, x}{2\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dy}{(x^2+y^2)}$$

Put $y = x \tan\theta$

$\therefore \; dy = x \sec^2\theta \, d\theta$

Also change limits,

$y \to -\infty \implies \theta = -\pi/2$

$y \to \infty \implies \theta = \pi/2$

Gateway Classes

Substitution

$$\therefore \quad E_x = \frac{\rho_s \, x}{2\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{x \sec^2\theta \cdot d\theta}{x^2 + x^2 \tan^2\theta}$$

$$= \frac{\rho_s \, x}{2\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{x \sec^2\theta \cdot d\theta}{x^2 (1 + \tan^2\theta)}$$

$$= \frac{\rho_s \, x}{2\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{x \sec^2\theta \cdot d\theta}{x^2 \sec^2\theta}$$

$$= \frac{\rho_s \, x}{2\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{d\theta}{x}$$

$$= \frac{\rho_s}{2\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} d\theta$$

$$= \frac{\rho_s}{2\pi\varepsilon_0} \left[\theta\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{\rho_s}{2\pi\varepsilon_0} \left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$\boxed{E_x = \frac{\rho_s}{2\varepsilon_0}} \qquad when \;\; x > 0$$

$$\boxed{E_x = -\frac{\rho_s}{2\varepsilon_0}} \qquad when \;\; x < 0$$

Gateway Classes

Electric Field Intensity due to Uniformly Charged Sphere

Derive an expression of Electric field intensity at all the possible location for a uniformly charged sphere.

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The electric field intensity E for a uniformly charged sphere can be derived using **Gauss's Law**, which states that

The expression depends on whether the charge is

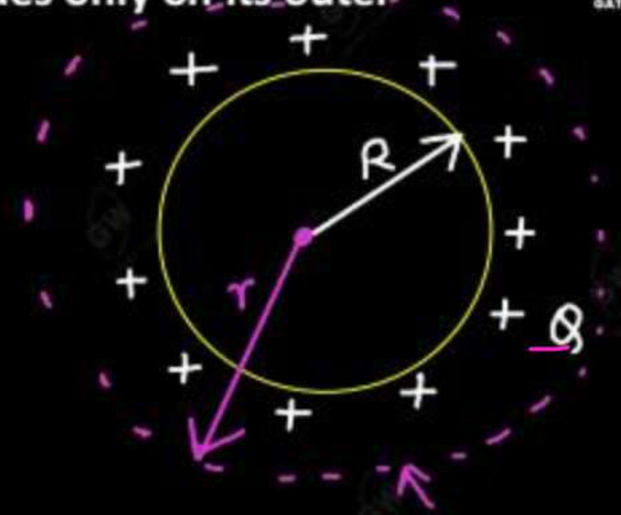
Case I :- **On the surface** (spherical shell / conductor) or

Case II :- **Throughout the volume** (non-conducting solid sphere).

Gateway Classes

Case 1: Uniformly Charged Spherical Shell (Conducting or Thin Hollow Sphere)

For a thin spherical shell of radius (R) carrying a total charge (Q), the charge resides only on its outer surface



Gaussian surface of radius r

1. Outside the sphere (r > R):
   A Gaussian sphere of radius (r) encloses all the charge Q .

$$E(4\pi r^2) = \frac{Q}{\epsilon_0} \quad \text{(Gauss Law)}$$

($4\pi r^2$: area of Gaussian surface)

$$\therefore \quad E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

2. On the surface of the sphere (r = R):

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

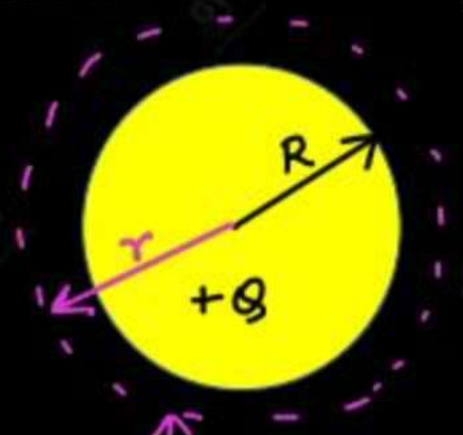
Gateway Classes

3. Inside the sphere (r < R):
   Since all charge resides on the outer surface, a Gaussian sphere inside encloses no charge . ($Q_{enclosed} = 0$)

$$E = 0$$

Case 2: Uniformly Charged Solid Sphere (Non-Conducting)

For a solid sphere of radius R and total charge Q, charge is distributed uniformly throughout its volume. The volume charge density is $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$ → charge, → volume of sphere



Gaussian surface of radius r

1. **Outside the sphere (r > R):**
   Similar to a point charge, the electric field is: $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

2. **On the surface of the sphere (r = R):**
   $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

3. **Inside the sphere (r < R):**
   A concentric Gaussian sphere of radius r encloses a fraction of the total charge.

$Q_{enclosed} = \rho \times$ volume of Gaussian surface $= \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = Q\frac{r^3}{R^3}$

Apply Gauss's Law: $E(4\pi r^2) = \frac{Q_{enclosed}}{\epsilon_0} = \frac{Q}{\epsilon_0}\frac{r^3}{R^3}$

$$\therefore \quad E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}$$

Gateway Classes

Que1) Point charges 1 mC and -2mC are located at (3,2,-1) and (-1,-1,4) ,respectively. Calculate the electric force on a 10nC charge located at (0,3,1) and the electric field intensity at that point.

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1\vec{r}_1}{|r_1|^3} + \frac{q_2\vec{r}_2}{|r_2|^3}\right] \quad -①$$

Now,

$$\vec{r}_1 = (0-3)\hat{a}_x + (3-2)\hat{a}_y + \{1-(-1)\}\hat{a}_z$$

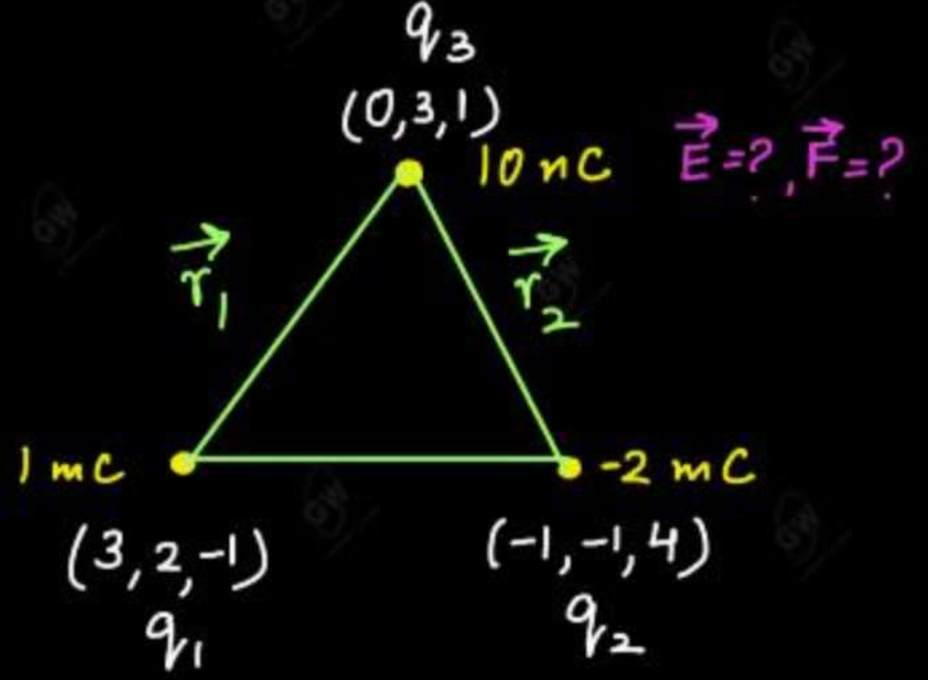
$$= -3\hat{a}_x + \hat{a}_y + 2\hat{a}_z$$

$$|\vec{r}_1| = \sqrt{(-3)^2+(1)^2+(2)^2} = \sqrt{14} = 3.7417\ m$$

$$\vec{r}_2 = \{0-(-1)\}\hat{a}_x + \{3-(-1)\}\hat{a}_y + \{1-4\}\hat{a}_z$$

$$= \hat{a}_x + 4\hat{a}_y - 3\hat{a}_z$$

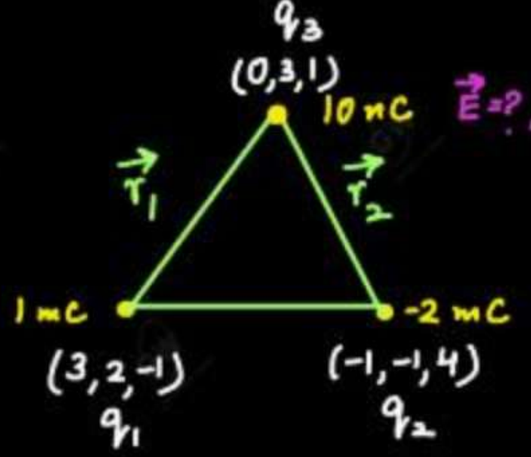
$$|\vec{r}_2| = \sqrt{(1)^2+(4)^2+(-3)^2} = \sqrt{26} = 5.0990\ m$$



Gateway Classes

Put in ①,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1\vec{r}_1}{|r_1|^3} + \frac{q_2\vec{r}_2}{|r_2|^3}\right]$$



$$= 9\times10^9\left[\frac{1\times10^{-3}(-3\hat{a}_x+\hat{a}_y+2\hat{a}_z)}{(3.7417)^3} + \frac{(-2\times10^{-3})(\hat{a}_x+4\hat{a}_y-3\hat{a}_z)}{(5.0990)^3}\right]$$

$$\approx (-650.7\,\hat{a}_x - 472.2\,\hat{a}_y + 758.2\,\hat{a}_z)\ \text{KV/m}\quad \text{Ans}$$

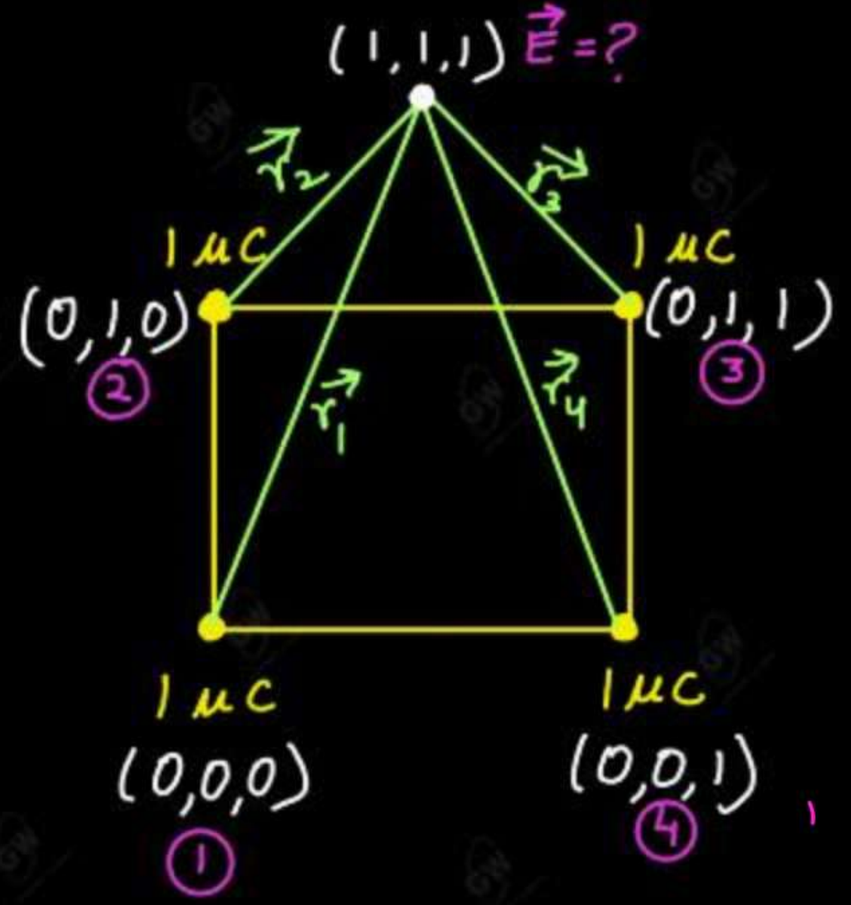
Now, $\vec{F} = q_3\vec{E}$

$$= (10\times10^{-9})(-650.7\,\hat{a}_x - 472.2\,\hat{a}_y + 758.2\,\hat{a}_z)$$

$$\approx -6.51\,\hat{a}_x - 4.72\,\hat{a}_y + 7.5\,\hat{a}_z\ \text{mN}\quad \text{Ans}$$

Gateway Classes

Que 2) Given four point charges of 1 μC each at (0,0,0), (0, 1,0), (0,1,1), and (0,0,1) respectively. Find E at (1,1,1).



Due to ①

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{(1\times10^{-6})[(1-0)\hat{a}_x + (1-0)\hat{a}_y + (1-0)\hat{a}_z]}{\left(\sqrt{(1-0)^2+(1-0)^2+(1-0)^2}\right)^3}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(1\times10^{-6})[\hat{a}_x + \hat{a}_y + \hat{a}_z]}{(\sqrt{3})^3}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(1\times10^{-6})[\hat{a}_x + \hat{a}_y + \hat{a}_z]}{3\sqrt{3}}$$

Gateway Classes

Due to ②

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{(1\times10^{-6})\left[(1-0)\hat{a}_x + (1-1)\hat{a}_y + (1-0)\hat{a}_z\right]}{\left(\sqrt{(1-0)^2+(1-0)^2}\right)^3}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(1\times10^{-6})\left[\hat{a}_x + \hat{a}_z\right]}{(\sqrt{2})^3}$$

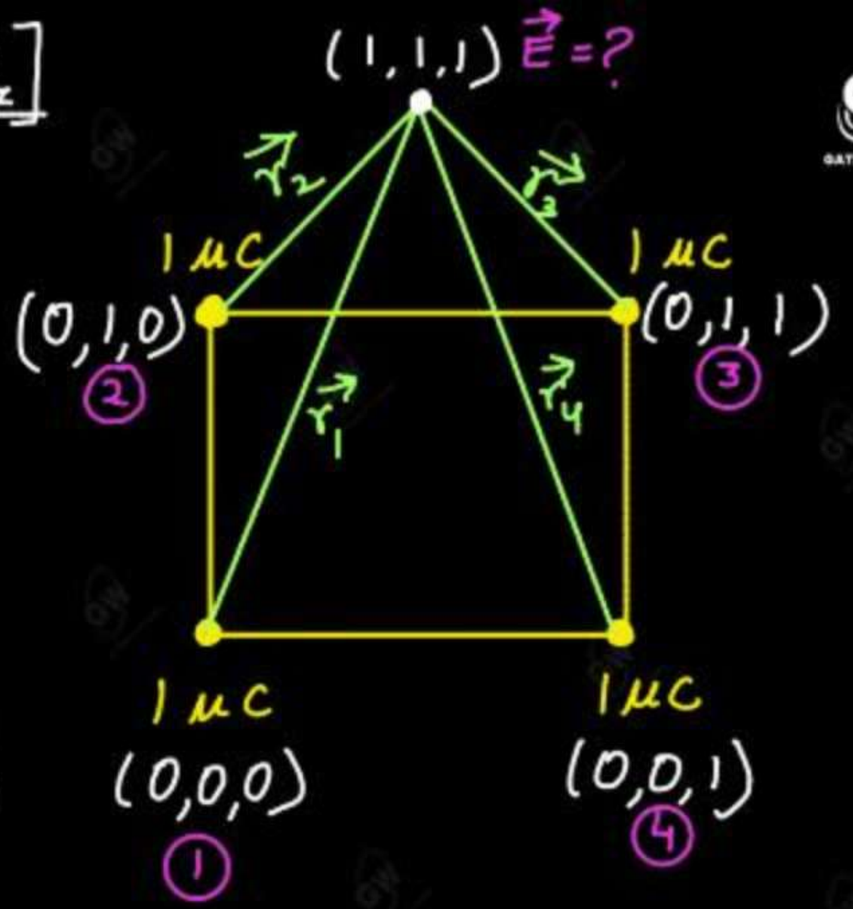
$$= \frac{1}{4\pi\epsilon_0} \frac{(1\times10^{-6})\left[\hat{a}_x + \hat{a}_z\right]}{2\sqrt{2}}$$

Due to ③

$$E_3 = \frac{1}{4\pi\epsilon_0} \frac{(1\times10^{-6})\left[(1-0)\hat{a}_x + (1-1)\hat{a}_y + (1-1)\hat{a}_z\right]}{\left(\sqrt{(1-0)^2}\right)^3}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(1\times10^{-6})\left[\hat{a}_x\right]}{(\sqrt{1})^3}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(1\times10^{-6})\left[\hat{a}_x\right]}{1}$$



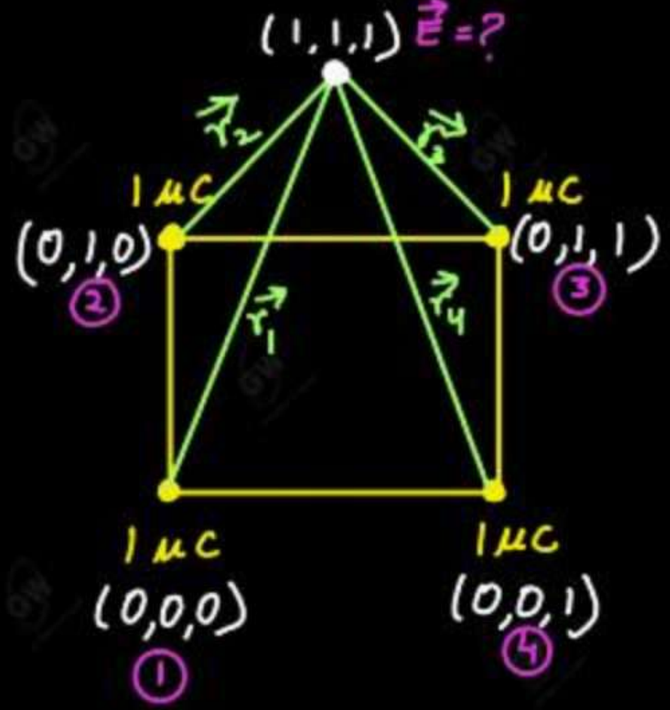
Gateway Classes

Due to (4)

$$E_4 = \frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[(1-0)\hat{a}_x+(1-0)\hat{a}_y+(1-1)\hat{a}_z]}{\left(\sqrt{(1-0)^2+(1-0)^2+(1-1)^2}\right)^3}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[\hat{a}_x+\hat{a}_y]}{(\sqrt{2})^3}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[\hat{a}_x+\hat{a}_y]}{2\sqrt{2}}$$



Add all,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[\hat{a}_x+\hat{a}_y+\hat{a}_z]}{3\sqrt{3}} + \frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[\hat{a}_x+\hat{a}_z]}{2\sqrt{2}} +$$

$$\frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[\hat{a}_x]}{1} + \frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[\hat{a}_x+\hat{a}_y]}{2\sqrt{2}}$$

Gateway Classes

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[\hat{a}_x+\hat{a}_y+\hat{a}_z]}{3\sqrt{3}} + \frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[\hat{a}_x+\hat{a}_z]}{2\sqrt{2}} +$$

$$\frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[\hat{a}_x]}{1} + \frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})[\hat{a}_x+\hat{a}_y]}{2\sqrt{2}}$$

Add component-wise (x, y, z)

$$E_x = \frac{1}{4\pi\epsilon_0}(1\times10^{-6})\left[\frac{1}{3\sqrt{3}} + \frac{1}{2\sqrt{2}} + \frac{1}{1} + \frac{1}{2\sqrt{2}}\right] = 17072.4 \text{ N/C or } 1.71\times10^{4} \text{ N/C}$$

$$E_y = \frac{1}{4\pi\epsilon_0}(1\times10^{-6})\left[\frac{1}{3\sqrt{3}} + 0 + 0 + \frac{1}{2\sqrt{2}}\right] = 4907.2 \text{ N/C or } 4.91\times10^{3} \text{ N/C}$$

$$E_z = \frac{1}{4\pi\epsilon_0}(1\times10^{-6})\left[\frac{1}{3\sqrt{3}} + \frac{1}{2\sqrt{2}} + 0 + 0\right] = 4907.2 \text{ N/C or } 4.91\times10^{3} \text{ N/C}$$

GW GATEWAY CLASSES
Gateway Classes

$$E = E_x + E_y + E_z$$

$$= 1.71 \times 10^4 \hat{a}_x + 4.91 \times 10^3 \hat{a}_y + 4.91 \times 10^3 \hat{a}_z$$

$$\text{Total } E = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

$$= \sqrt{(17072.4)^2 + (4907.2)^2 + (4907.2)^2}$$

$$= 18429 \text{ N/C}$$ Ans

Gateway Classes

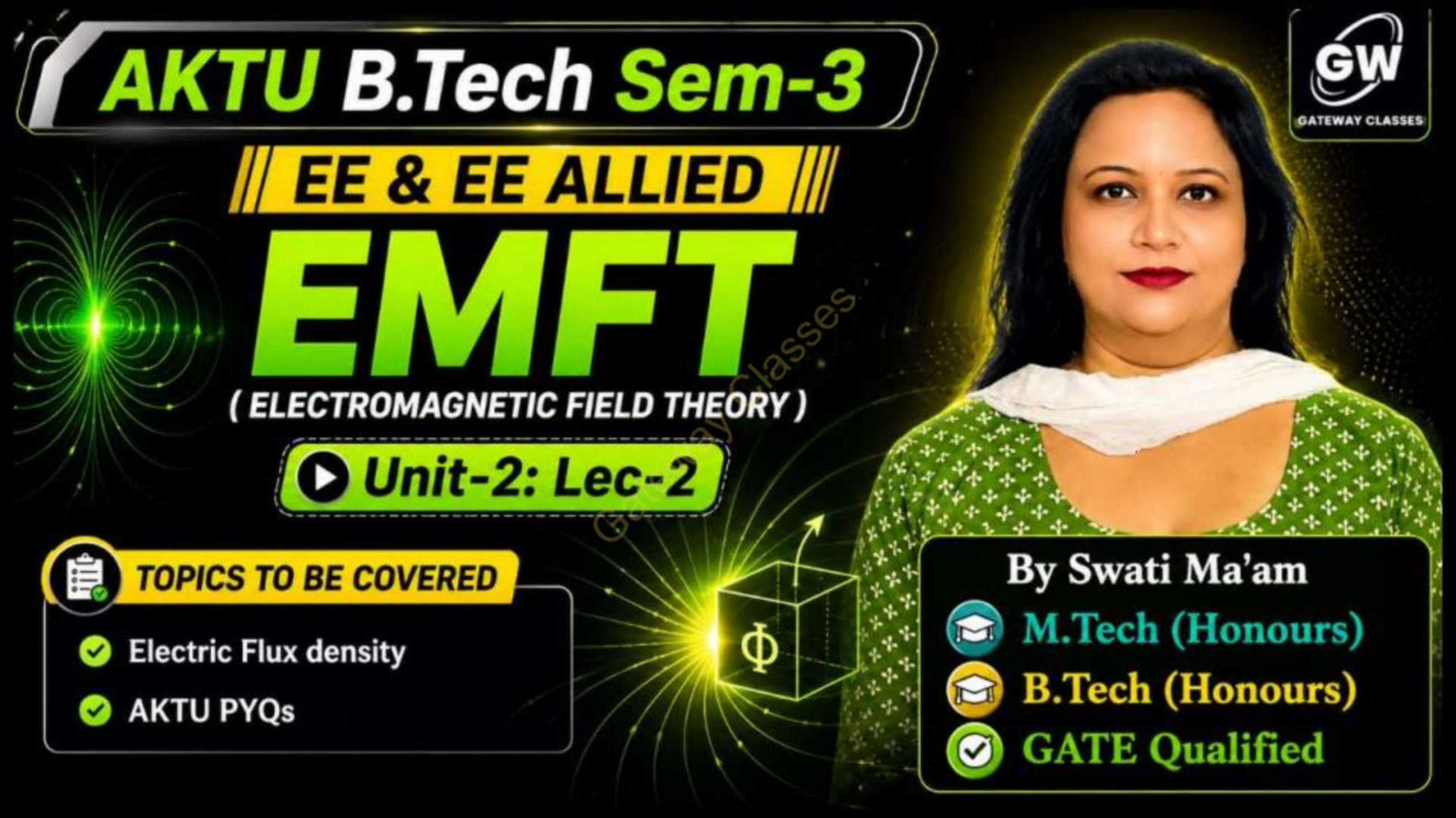
AKTU B.Tech Sem-3

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit-2: Lec-2



GW GATEWAY CLASSES

TOPICS TO BE COVERED

- Electric Flux density
- AKTU PYQs

GatewayClasses

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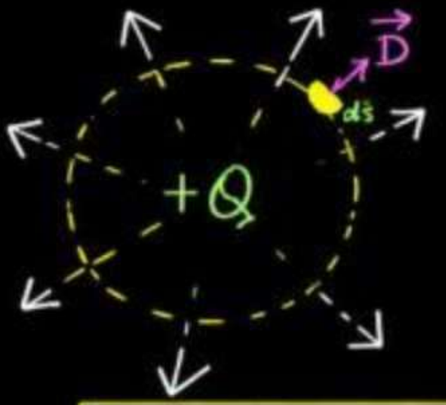
- B.Tech (Honours)
- GATE Qualified

AKTU PYQ-2022-23

Electric Flux Density $\vec{D}$

Electric Flux Density or electric displacement D is a vector field which measures the amount of electric flux passing through a unit area , representing the density of electric field lines.

$$\vec{D} = \epsilon_0 \vec{E}$$



$$\Psi = \iint \vec{D} \cdot d\vec{s}$$

Total flux over closed surface.

It is a vector quantity whose direction is always taken normal to the surface.

Unit :- coulomb / meter^2

Due to a point charge

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

$$\underbrace{\epsilon_0 \vec{E}}_{\vec{D}} = \frac{Q}{4\pi r^2} \hat{a}_r$$

Gateway Classes

$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r$$

In Dielectric Medium $\vec{D} = \varepsilon \vec{E}$ General.

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} \quad (\varepsilon = \varepsilon_0 \varepsilon_r)$$

In Free space, $\varepsilon_r = 1$

$\therefore$ $\vec{D} = \varepsilon_0 \vec{E}$

Gateway Classes

GW
GATEWAY CLASSES

$$\vec{D} = \varepsilon_0 \vec{E} = \varepsilon_0 \int \frac{\rho_L \, dL}{4\pi \varepsilon_0 r^2} \hat{a}_r$$

D due to line charge distribution

$$\vec{D} = \int \frac{\rho_L \, dL}{4\pi r^2} \hat{a}_r$$

D due to surface charge distribution

$$\vec{D} = \iint \frac{\rho_s \, dS}{4\pi r^2} \hat{a}_r$$

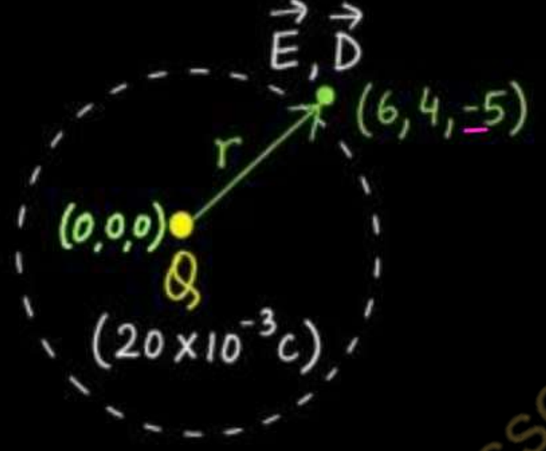
D due to volume charge distribution

$$\vec{D} = \iiint \frac{\rho_v \, dV}{4\pi r^2} \hat{a}_r$$

Gateway Classes

Que- Find the flux density at a point (6 ,4,-5) caused by a point charge of 20 mC at the origin.

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r$$



where $\hat{a}_r = \dfrac{\vec{r}}{|\vec{r}|}$

$$\vec{r} = 6\hat{a}_x + 4\hat{a}_y - 5\hat{a}_z$$

$$|\vec{r}| = \sqrt{(6)^2 + (4)^2 + (-5)^2}$$

$$= \sqrt{77}$$

$$\therefore \quad r^2 = 77$$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r}$$

$$\vec{D} = \frac{20\times10^{-3}}{4\times3.14\times77}\cdot\frac{6\hat{a}_x + 4\hat{a}_y - 5\hat{a}_z}{\sqrt{77}}$$

$$\vec{D} = \left(1.41\,\hat{a}_x + 0.94\hat{a}_y - 1.18\,\hat{a}_z\right)$$

$\mu C/m^2$

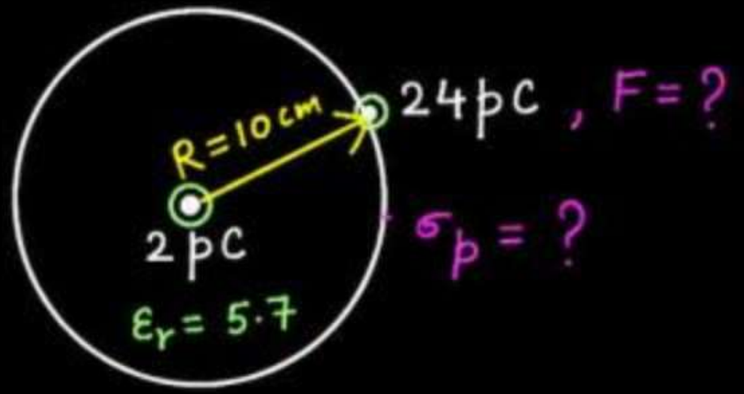
Ans

Gateway Classes

Que i) A dielectric sphere ($\varepsilon_r = 5.7$) of radius 10 cm has a point charge of 2pC placed at its center. Calculate

i. The surface density of polarization charge on the surface of the sphere.

ii. The force exerted by the charge on a 24pC point charge placed on the sphere.



(i) Surface charge density σ_p on the surface of sphere is

$$\sigma_p = \frac{q_p}{A}$$

where

q_p → total polarization charge

$$q_p \rightarrow -q\left(1 - \frac{1}{\varepsilon_r}\right)$$

↳ A → is surface area of sphere

↳ $4\pi R^2$

Gateway Classes

Now,

$$q_p = -q\left(1 - \frac{1}{\epsilon_r}\right)$$

$$= -2 \times 10^{-12}\left[1 - \frac{1}{5.7}\right]$$

$$= -1.649 \times 10^{-12}\ C.$$

$$A = 4\pi R^2$$

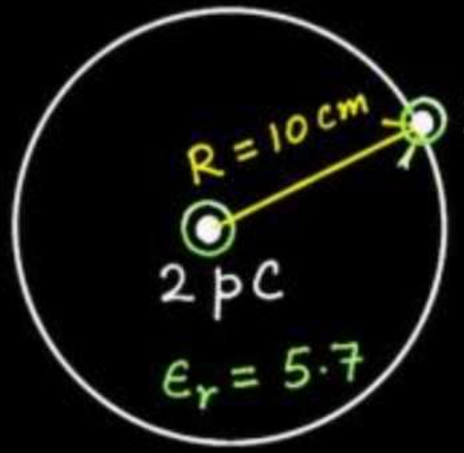
$$= 4 \times 3.14 \times (0.1)^2$$

$$= 0.1257\ m^2$$

$$\therefore\ \sigma_p = \frac{-1.649 \times 10^{-12}}{0.1257}$$

$$= -1.31 \times 10^{-11}\ C/m^2$$

Ans



$F = ?$

$\sigma_p = ?$

(i) $$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \cdot \frac{q_1 q_2}{R^2}$$

Note

$$= \frac{1 \times 2 \times 10^{-12} \times 24 \times 10^{-12}}{4 \times 3.14 \times 8.854 \times 10^{-12} \times 5.7 \times (0.1)^2}$$

$$= 7.58 \times 10^{-12}\ N$$

Ans

Gateway Classes

Que 2) Given the potential $V = \frac{560}{3r^2} \sin 2\theta \cos\phi$, Find the Electric Flux density D at $(2, 90^0, 0)$.

Also calculate the work done in moving a 10 μC charge from point A$(1, 30^0, 120^0)$ to B$(2, 60^0, 30^0)$.

(i) V → E → D

$\vec{E} = -\vec{\nabla} \cdot V$

$\vec{D} = \epsilon_0 \vec{E}$

Now, $V = \frac{560}{3r^2} \sin 2\theta \cos\phi$

$$E_r = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{560}{3r^2} \sin 2\theta \cos\phi\right)$$

$$= +\frac{1120}{3r^3} \sin 2\theta \cos\phi$$

$$E_\theta = -\frac{1}{r} \cdot \frac{\partial V}{\partial \theta} = -\frac{1}{r} \cdot \frac{\partial}{\partial \theta}\left(\frac{560}{3r^2} \sin 2\theta \cos\phi\right)$$

$$= -\frac{560}{3r^3}\left[2\cos 2\theta \cos\phi\right]$$

$$= -\frac{1120}{3r^3} \cos 2\theta \cos\phi$$

Gateway Classes

$$E_\phi = \frac{-1}{r\sin\theta}\frac{\partial V}{\partial \phi} = -\frac{-1}{r\sin\theta}\frac{\partial}{\partial \phi}\left(\frac{560}{3r^2}\sin 2\theta\cos\phi\right)$$

$$= \frac{-1}{r\sin\theta}\left[-\frac{560}{3r^2}(\sin 2\theta \sin\phi)\right]$$

$$= \frac{560}{3r^3\sin\theta}\left[2\sin\theta\cos\theta\sin\phi\right]$$

$$= \frac{1120}{3r^3}\cos\theta\sin\phi$$

At point $(2, 90^\circ, 0)$

$r \quad \theta \quad \phi$

$$E_r = \frac{1120}{3r^3}\sin 2\theta\cos\phi$$

$$= \frac{1120}{3(2)^3}\underbrace{\sin 180^\circ}_{0}\ \underbrace{\cos 0^\circ}_{1} = 0$$

$$E_\theta = -\frac{1120}{3r^3}\cos 2\theta\cos\phi$$

$$= -\frac{1120}{3(2)^3}\underbrace{\cos 180^\circ}_{-1}\ \underbrace{\cos 0^\circ}_{1}$$

$$= \frac{1120}{24} = 46.67 \text{ V/m}$$

$$E_\phi = \frac{1120}{3r^3}\cos\theta\sin\phi$$

$$= \frac{1120}{3(2)^3}\underbrace{\cos 90^\circ}_{0}\ \underbrace{\sin 0^\circ}_{0} = 0$$

$$\therefore \vec{E} = 46.67\ \hat{a}_\theta$$

Now, $\vec{D} = \varepsilon_0 \vec{E}$

$$= (8.854 \times 10^{-12})(46.67\ \hat{a}_\theta)$$

$$\therefore \boxed{\vec{D} = 413.2\ \hat{a}_\theta\ pC/m^2}\ \text{Ans.}$$

Gateway Classes

(ii) Work done

$$W = q(V_B - V_A)$$

Point A $(1, 30^0, 120^0)$ → r, θ, ϕ

$$V_A = \frac{560}{3r^2} \sin 2\theta \cos\phi$$

$$= \frac{560}{3(1)^2} \underbrace{\sin 60^0}_{\frac{\sqrt{3}}{2}} \underbrace{\cos 120^0}_{-\frac{1}{2}}$$

$$= -80.83 \text{ V}$$

Point B $(2, 60^0, 30^0)$ → r, θ, ϕ

$$V_B = \frac{560}{3r^2} \sin 2\theta \cos\phi$$

$$= \frac{560}{3(2)^2} \underbrace{\sin 120^0}_{\frac{\sqrt{3}}{2}} \underbrace{\cos 30^0}_{\frac{\sqrt{3}}{2}}$$

$$= 35 \text{ V}$$

$$\therefore W = q(V_B - V_A)$$

$$= 10 \times 10^{-6} \left[35 - (-80.83)\right]$$

$$= 1.158 \text{ mJ}$$ Ans

Gateway Classes

AKTU B.Tech Sem-3

GATEWAY CLASSES

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit-2: Lec-3

Topics to be covered

- Gauss Law
- Applications of Gauss Law
- Electric Potential
- PYQs

Gateway Classes

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- B.Tech (Honours)
- GATE Qualified

GW GATEWAY CLASSES

Gauss Law (for Electrostatics)

AKTU PYQ-2022-23,2019-20

GW GATEWAY CLASSES

It states that the total electric flux passing through any closed surface is equal to the net charge enclosed by that surface.

$$\Psi = \oint_s \vec{D} \cdot \vec{ds} = Q \quad -(1)$$

$Q \rightarrow$ charge enclosed.

Ψ :- Electric flux through a closed surface having volume V with a charge density ρ (C/m^3)

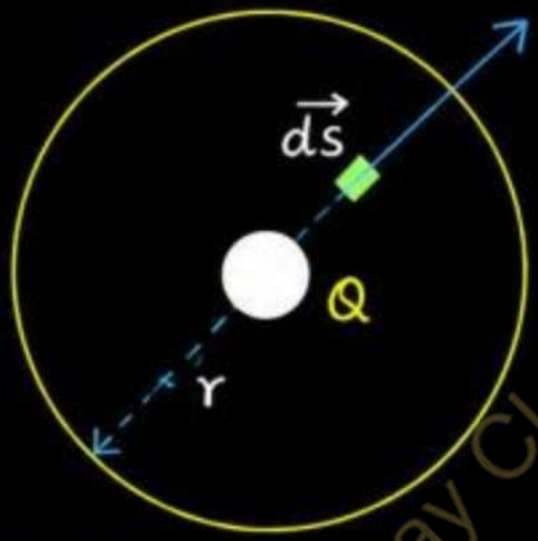
$$\Psi = \int_v \rho \, dV \quad -(2)$$

$$\therefore \quad \Psi = \oint_s \vec{D} \cdot \vec{ds} = Q = \int_v \rho \, dV \qquad \text{From (1) \& (2)}$$

Gateway Classes

Proof :-

Consider the surface of a sphere of radius r enclosing a charge Q at its center.



$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r$$

$$\vec{D} = \frac{Q}{4\pi r^2}\cdot\hat{a}_r$$

Gateway Classes

In Spherical Coordinate System,

$$\vec{ds}_r = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$\therefore \quad \psi = \oint_s \vec{D} \cdot \vec{ds}$$

$$= \oint_s \left(\frac{Q}{4\pi r^2}\,\hat{a}_r\right)\cdot\left(r^2 \sin\theta \, d\theta \, d\phi\right)\hat{a}_r$$

$$\psi = \oint_S \left(\frac{Q}{4\pi r^2}\,\hat{a}_r\right)\left(r^2 \sin\theta\, d\theta\, d\phi\right)\hat{a}_r$$

$$= \frac{Q}{4\pi}\oint_S \sin\theta\, d\theta\, d\phi \qquad (\hat{a}_r \cdot \hat{a}_r = 1)$$

$$= \frac{Q}{4\pi}\int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$= \frac{Q}{4\pi}\left[-\cos\theta\right]_0^{\pi}\left[\phi\right]_0^{2\pi}$$

$$= -\frac{Q}{4\pi}\left[\cos\pi - \cos 0\right]\left[2\pi - 0\right]$$

$$= -\frac{Q}{4\pi}\left[-1-1\right]\left[2\pi\right]$$

$$= \frac{Q}{4\pi}(2)(2\pi)$$

$$= Q$$

$$\boxed{\oint \vec{D}\cdot d\vec{s} = Q}$$

Hence Proved.

Gateway Classes

Applications of Gauss's Law

To find E using Gauss Law

Steps :-

1. Assume Gaussian surface (Point Charge -> Spherical , Line charge -> Cylindrical)
2. Apply Gauss Law

   $\Psi = Q_{enclosed}$
3. $\vec{D} = \left(\frac{\Psi}{A}\right)\hat{a}_r$

   where A is area of Gaussian Surface
4. Put Ψ and A in (3) , find $\vec{D}$.
5. Find E by

$$\vec{D} = \varepsilon_o \vec{E}$$

Gateway Classes

Applications of Gauss's Law

1. Assume Gaussian surface
2. Apply Gauss Law

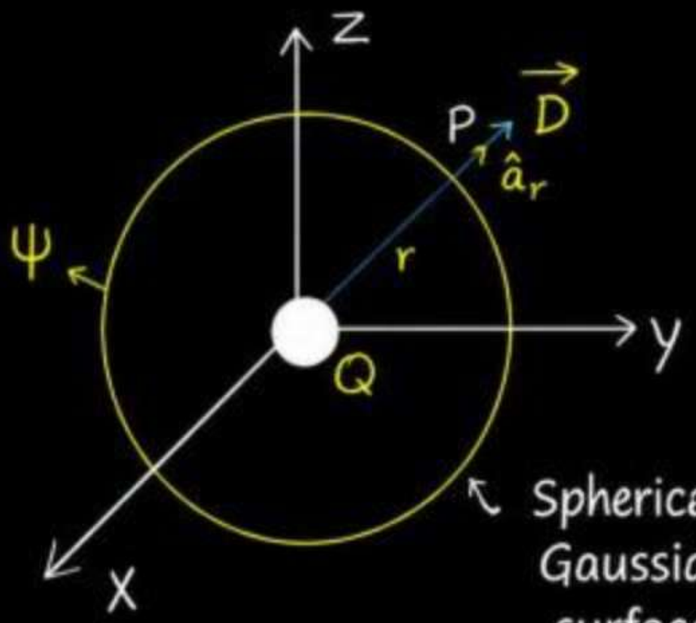
$$\Psi = \frac{Q_{enclosed}}{A}$$

3. $D = (\frac{\Psi}{A}), \hat{a}_r$
4. Put Ψ and A in (3) , find D.
5. Find E by $D = \varepsilon_0 E$

1. To find E due to a point charge at a point P

We draw a Gaussian surface (Spherical surface) enclosing point charge Q and passing through point P

The electric flux Ψ coming out of Gaussian surface is given by



↖ Spherical Gaussian surface

$$\Psi = Q \quad \text{(By Gauss Law)}$$

Now,

$$\vec{D} = \frac{\Psi}{A}\,\hat{a}_r$$

→ Unit vector along the radius r.

$A \rightarrow 4\pi r^2$

$$\therefore \quad \vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r$$

Gateway Classes

$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r \quad -①$$

Also,

$$\vec{D} = \varepsilon_o \vec{E} \quad -②$$

Put ① in ②

$$\varepsilon_o \vec{E} = \frac{Q}{4\pi r^2}\,\hat{a}_r$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_o r^2}\,\hat{a}_r}$$

Gateway Classes

Applications of Gauss's Law

1. Assume Gaussian surface
2. Apply Gauss Law
$$\Psi = Q_{enclosed}$$
3. $D = (\frac{\Psi}{A})\hat{a}_r$ $\quad A$
4. Put Ψ and A in (3) , find D
5. Find E by $D = \varepsilon_0 E$

2. To find E due to an infinite line charge at a point P

We draw a Gaussian surface (Cylindrical surface) enclosing *line charge* and passing through point P.

We have to calculate $\vec{E}$ at a distance r. We consider a cylindrical Gaussian surface of radius r, and length L.

The total charge enclosed in the Gaussian surface is given by

$$Q = \int \rho_\ell \, dl = \rho_\ell \, L \quad -(1)$$

Now , according to Gauss Law,

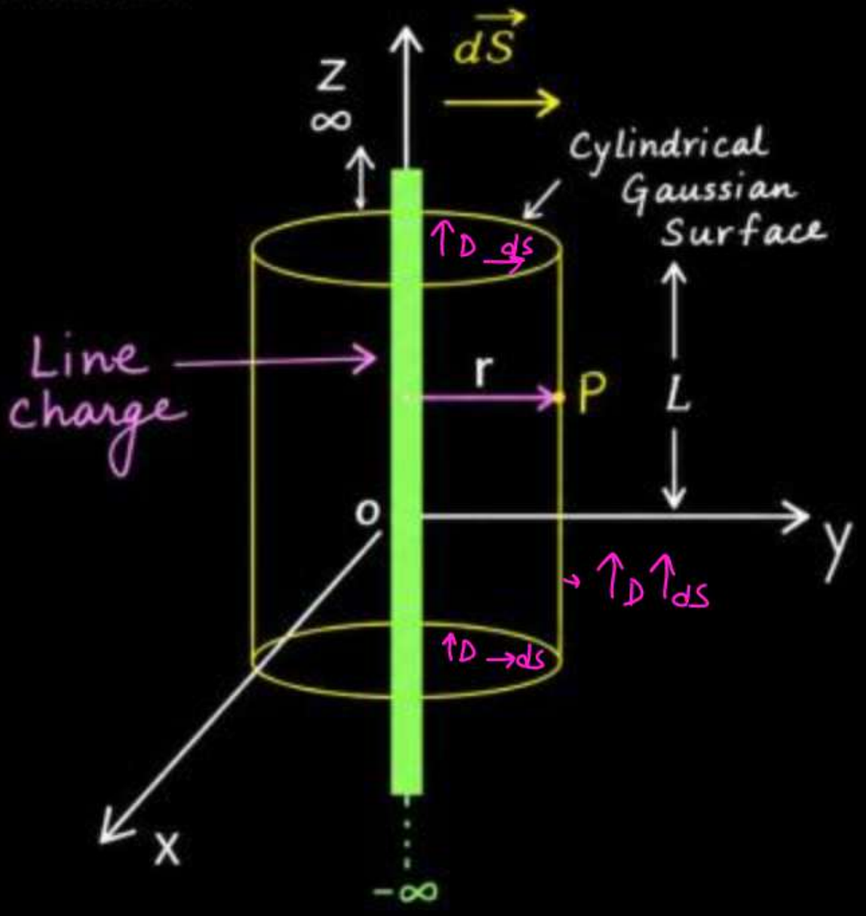
$$q = \oint_S \vec{D} \cdot \vec{dS} \quad -(2)$$

Cylindrical surface has 3 surfaces

Upper surface

Side Curved Surface

Bottom Surface



Gateway Classes

$$\oint_S \vec{D} \cdot \vec{dS} = \underset{top}{\iint} D\, dS \underbrace{\cos 90^\circ}_{0} + \underset{sides}{\iint} D \underbrace{dS}_{\rho\, d\phi\, dz \text{ or } r\, d\phi\, dz} \underbrace{\cos 0^\circ}_{1} + \underset{bottom}{\iint} D\, dS \underbrace{\cos 90^\circ}_{0}$$

here $\rho\, d\phi\, dz$ or $r\, d\phi\, dz$ } Note

$$= \iint D\, r\, d\phi\, dz$$

$$= D r \int_0^{2\pi} d\phi \int_0^{L} dz$$

$$= D r\, (2\pi)\, (L)$$

$\therefore$ From (2), $Q = D r\, (2\pi)\, (L)$

From (1), $\rho_L L = D r\, (2\pi)\, (L)$

$\therefore$ $$\boxed{D = \frac{\rho_L}{2\pi r}}$$

In Vector Form,

$$\boxed{\vec{D} = \frac{\rho_L}{2\pi r}\, \hat{a}_r}$$

and

$$\therefore \quad \vec{D} = \varepsilon_0 \vec{E}$$

$\therefore$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi \varepsilon_0 r}\, \hat{a}_r}$$

Gateway Classes

IInd Method

By Gauss Law,

$$\psi = Q_{enclosed} = \rho_l \cdot L$$

The electric flux density at point P is

$$\vec{D} = \frac{\psi}{A}\,\hat{a}_r$$

$\psi \rightarrow \rho_l \cdot L$

$A \rightarrow 2\pi r\, L$

$$\vec{D} = \frac{\rho_l \cdot L}{2\pi r L}\,\hat{a}_r$$

$$\boxed{\vec{D} = \frac{\rho_l}{2\pi r}\,\hat{a}_r}$$

and

$$\therefore \quad \vec{D} = \varepsilon_0 \vec{E}$$

$$\therefore \quad \boxed{\vec{E} = \frac{\rho_l \cdot \hat{a}_r}{2\pi \varepsilon_0 r}}$$

Gateway Classes

Electric Potential

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As Liquid flows from higher level to a lower level ,similarly Electric charge flows from a region of higher potential to a region of lower potential.

Consider that q_0 is moved from point A to another point B by an external agent , therefore some work has to be done.

$$V_{AB} = V_B - V_A = \frac{\text{Work done}}{\text{Magnitude of test charge}} = \frac{W}{q_0}$$

Case

If point A is chosen at infinity , then V_A is taken as zero, then

$$V_B = \frac{W}{q_0}$$

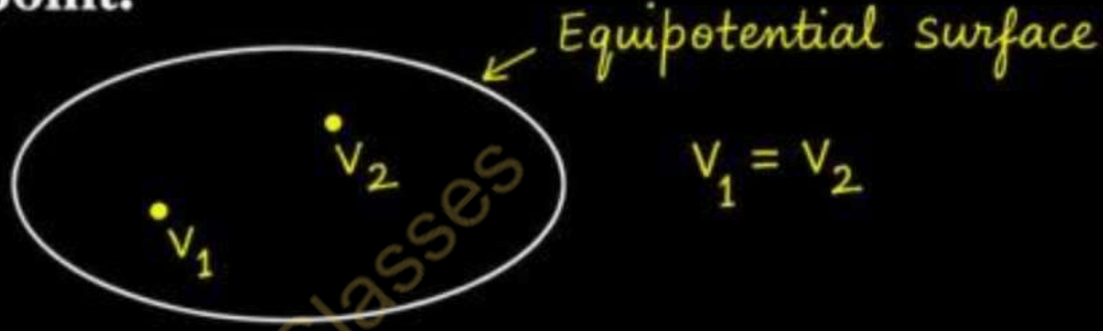
→ Definition

Electric potential at a point in electric field is defined as the work done by an external agent in carrying a unit positive test charge from infinity to that point against the electric force of the field.

Gateway Classes

Equipotential surfaces

Equipotential Surface in an electric field is a surface on which the potential is same at every point.



The lines of force at every point of the equipotential surface are perpendicular to the surface.

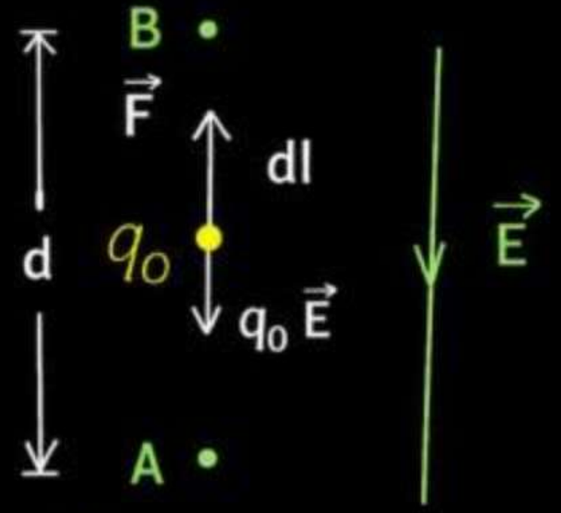
The work done in moving a charge on the equipotential surface is zero.

work done = potential difference × charge
= 0 × charge
= zero.

Gateway Classes

Relation between V and $\vec{E}$

Let the positive test charge q_0 is moved by an external agent through a small distance dl in the opposite direction of $\vec{E}$.



Work done $$dW = -\vec{F} \cdot d\vec{l} = -q_0 \vec{E} \cdot d\vec{l}$$

Work is being done by an external agent in opposite direction of $\vec{E}$.

Total work done in carrying q_0 from A to B

$$W = \int_A^B -q_0 \vec{E} \cdot d\vec{l} = -q_0 \int_A^B \vec{E} \cdot d\vec{l}$$

$$V_{AB} = V_B - V_A = \frac{W_{AB}}{q_0} = -\int_A^B \vec{E} \cdot d\vec{l}$$

If A is taken at infinity then $V_A = 0$

$$\boxed{V_B = -\int_\infty^B \vec{E} \cdot d\vec{l}}$$

Electric potential at a point in the electric field can be expressed as a line integral of the electric field.

Gateway Classes

Potential difference between two points

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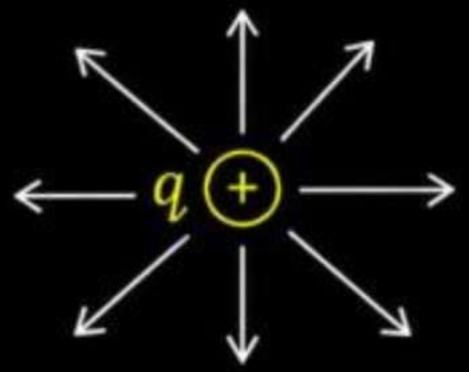
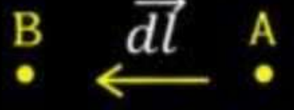
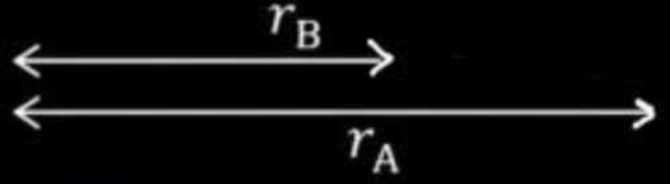






$\vec{dl}$ is small length element between points A & B.

$$V_{AB} = -\int_{A}^{B} \vec{E} \cdot \vec{dl} = -\int_{r_A}^{r_B} \vec{E} \cdot \vec{dl}$$

where $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r \quad , \quad \vec{dl} = dr\,\hat{a}_r$

$$\therefore \quad V_{AB} = -\int_{r_A}^{r_B} \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r\,dr \cdot \hat{a}_r$$

$$V_{AB} = -\int_{r_A}^{r_B} \frac{q}{4\pi\varepsilon_0 r^2}\,dr \qquad \because \hat{a}_r \cdot \hat{a}_r = 1$$

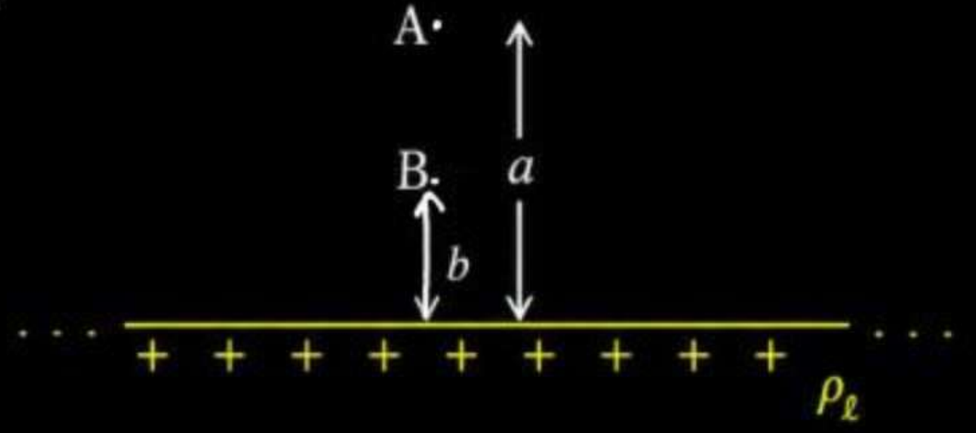
$$= -\frac{q}{4\pi\dot{\varepsilon}_0}\int_{r_A}^{r_B} \frac{1}{r^2}\,dr$$

$$= -\frac{q}{4\pi\dot{\varepsilon}_0}\left[-\frac{1}{r}\right]_{r_A}^{r_B}$$

$$\boxed{V_{AB} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]}$$

Gateway Classes

Potential Difference due to Uniformly Charged Line



$$V_{AB} = -\int_a^b \vec{E}\cdot d\vec{l} = -\int_a^b \vec{E}\cdot d\vec{l}$$

where $\vec{E} = \dfrac{\rho_\ell}{2\pi\epsilon_0 r}\,\hat{a}_r$, $d\vec{l} = dr\,\hat{a}_r$

$$\therefore\ V_{AB} = -\int_a^b \frac{\rho_\ell}{2\pi\epsilon_0 r}\,\hat{a}_r\,dr\,\hat{a}_r$$

$$V_{AB} = -\int_a^b \frac{\rho_\ell}{2\pi\epsilon_0 r}\,dr \qquad \therefore\ \hat{a}_r\cdot\hat{a}_r = 1$$

$$= -\frac{\rho_\ell}{2\pi\epsilon_0}\int_a^b \frac{1}{r}\,dr$$

$$= -\frac{\rho_\ell}{2\pi\epsilon_0}\left[\log_e r\right]_a^b$$

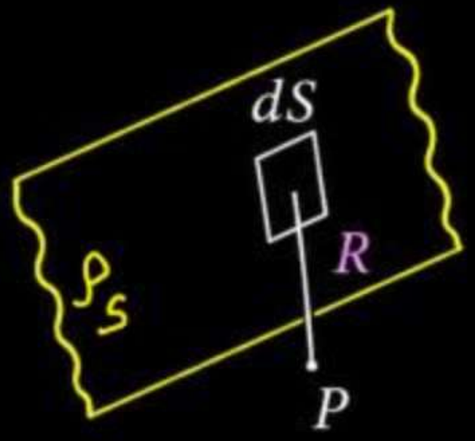
$$= -\frac{\rho_\ell}{2\pi\epsilon_0}\left[\log_e b - \log_e a\right]$$

$$= \frac{\rho_\ell}{2\pi\epsilon_0}\left[\log_e a - \log_e b\right]$$

$$\boxed{V_{AB} = \frac{\rho_\ell}{2\pi\epsilon_0}\log_e\left(\frac{a}{b}\right)}$$

Gateway Classes

Potential Difference due to Surface Charged Distribution



dS is small differential area.

Potential at $P = ?$

$$dq = \rho_s \, dS$$

(charge on dS)

$$dV = \frac{dq}{4\pi\epsilon_0 R} = \frac{\rho_s \, dS}{4\pi\epsilon_0 R}$$

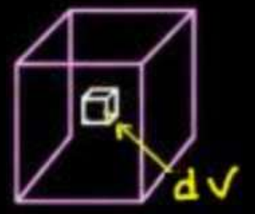
Total Potential on entire surface

$$V = \int_S \frac{\rho_s \, dS}{4\pi\epsilon_0 R}$$

$$V = \frac{1}{4\pi\epsilon_0} \int_S \frac{\rho_s \, dS}{R}$$

If Volume charge distribution,

$$V = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho_v \, dV}{R}$$



Gateway Classes

Potential Difference due to Charged Disc

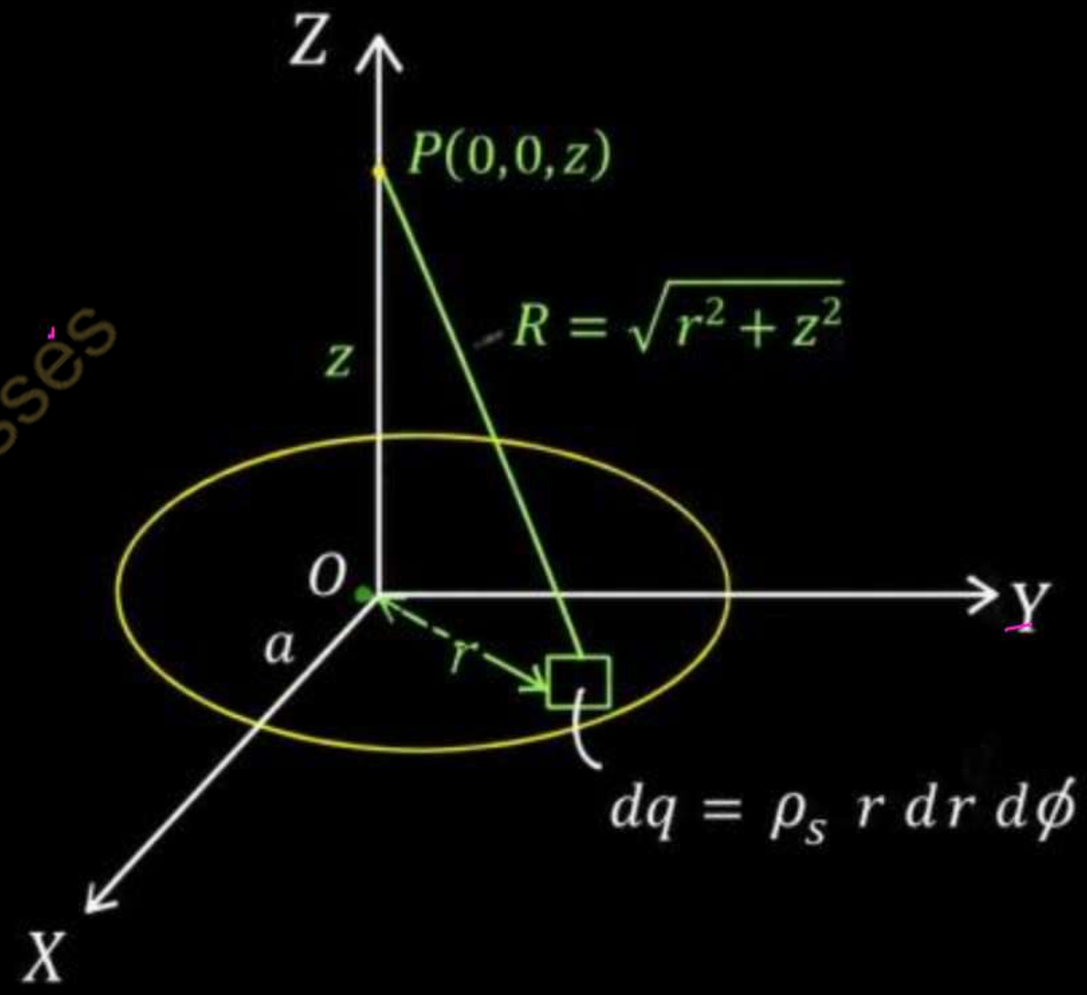
$$V = \int_s \frac{dq}{4\pi\varepsilon_0 R} \quad -①$$

Now, $dq = \rho_s \, r \, dr \, d\phi$ and $R = \sqrt{r^2 + z^2}$

Put in ①,

$$V = \int_s \frac{\rho_s \, r \, dr \, d\phi}{4\pi\varepsilon_0 \sqrt{r^2 + z^2}}$$

$$= \frac{\rho_s}{4\pi\varepsilon_0} \int_{r=0}^{a} \frac{r \, dr}{\sqrt{r^2 + z^2}} \int_{\phi=0}^{2\pi} d\phi \quad -②$$



Gateway Classes

$$= \frac{\rho_s}{4\pi\varepsilon_0} \int_{r=0}^{a} \frac{r\,dr}{\sqrt{r^2+z^2}} \int_{\phi=0}^{2\pi} d\phi \quad - (2)$$

Now, $\int_{\phi=0}^{2\pi} d\phi = [\phi]_0^{2\pi} = 2\pi$

For $\int_{r=0}^{a} \frac{r\,dr}{\sqrt{r^2+z^2}}$

Put $r^2 + z^2 = p^2$

$2r\,dr = 2p\,dp$

$r\,dr = p\,dp$

Changing limits

when $r = 0$, $p = z$

$r = a$, $p = \sqrt{a^2+z^2}$

Substitute in

$$V = \frac{\rho_s}{4\pi\varepsilon_0} \cdot 2\pi \int_{r=0}^{a} \frac{r\,dr}{\sqrt{r^2+z^2}}$$

$$V = \frac{\rho_s}{4\pi\varepsilon_0} \cdot 2\pi \int_{p=z}^{\sqrt{a^2+z^2}} \frac{p\,dp}{p}$$

$$= \frac{\rho_s}{4\pi\varepsilon_0} \cdot 2\pi \int_{p=z}^{\sqrt{a^2+z^2}} dp$$

$$\mathrm{V} = \frac{\rho_s}{2\varepsilon_0} \left[\sqrt{a^2+z^2} - z \right]$$

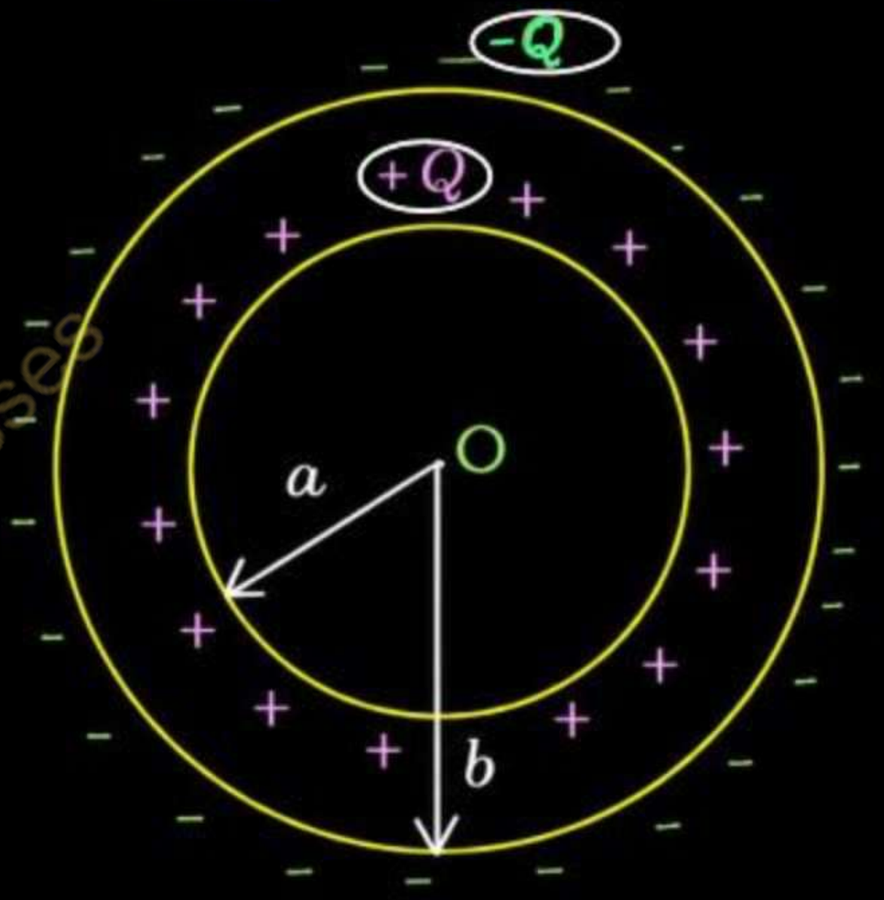
Gateway Classes

Potential between two conducting spherical shells

+ Q on inner surface
- Q on outer surface

a - inner radius
b - outer radius

V_{ba} = ?



Potential difference between the shells

$$V_{ba} = -\int_{b}^{a} \vec{E} \cdot d\vec{r}$$

$$\text{where} \quad E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Gateway Classes

$$\therefore \quad V_{ba} = -\int_{b}^{a} \frac{Q}{4\pi\varepsilon_0 r^2}\, dr$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{b}^{a}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{b}^{a}$$

Gateway Classes

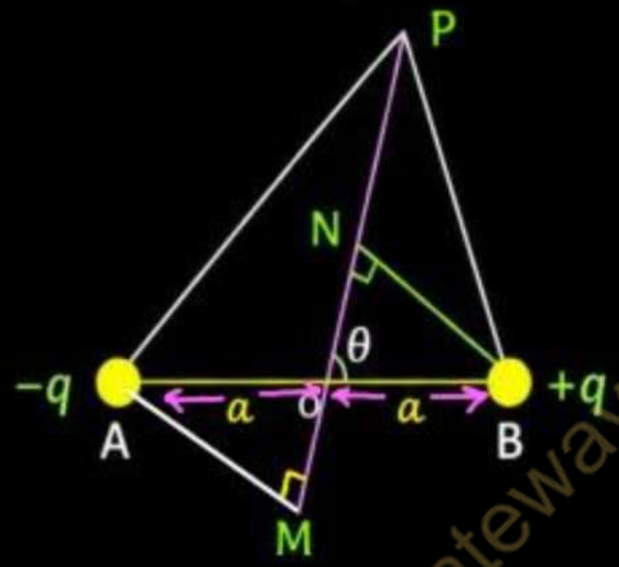
$$\boxed{V_{ba} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]}$$

Potential due to dipole

Electric dipole :- When two charges equal in magnitude but opposite in sign and separated by a small distance the system is called an Electric Dipole.

AKTU PYQ-2025-26, 2021-22

Narrate the concept of electric dipole moment.



$OP = r$, $AB = 2a$

Potential at point P = ?

$BP = NP$

$AP = MP$

$ON = OM = a\cos\theta$

Gateway Classes

Charges −q at A and +q at B , separation 2a , point P at distance r from midpoint O with angle θ

Dipole moment $p = 2aq$

distance between two charges = 2a

Potential at P due to +q at B

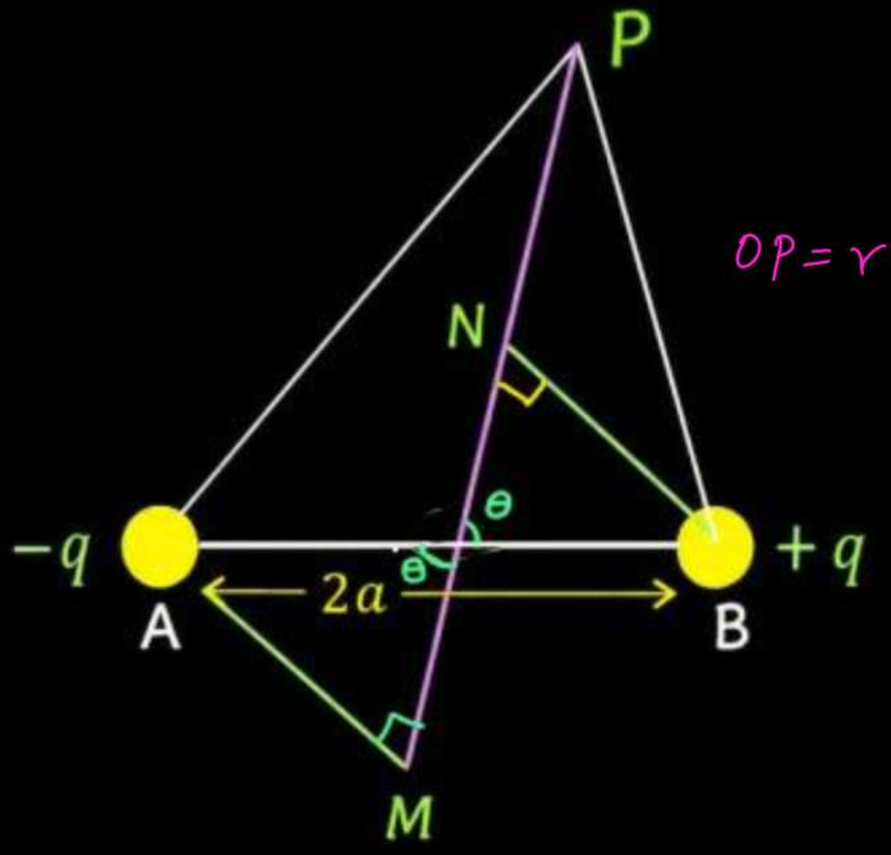
$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{BP}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{NP}\right)$$

Potential at P due to +q at A

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{AP}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{MP}\right)$$

Resultant Potential $V = V_1 + V_2$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{NP} + \frac{(-q)}{MP}\right)$$



$BP = NP$

$AP = MP$

$ON = OM = a\cos\theta$

Gateway Classes

Potential at P due to +q at B

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{BP}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{NP}\right)$$

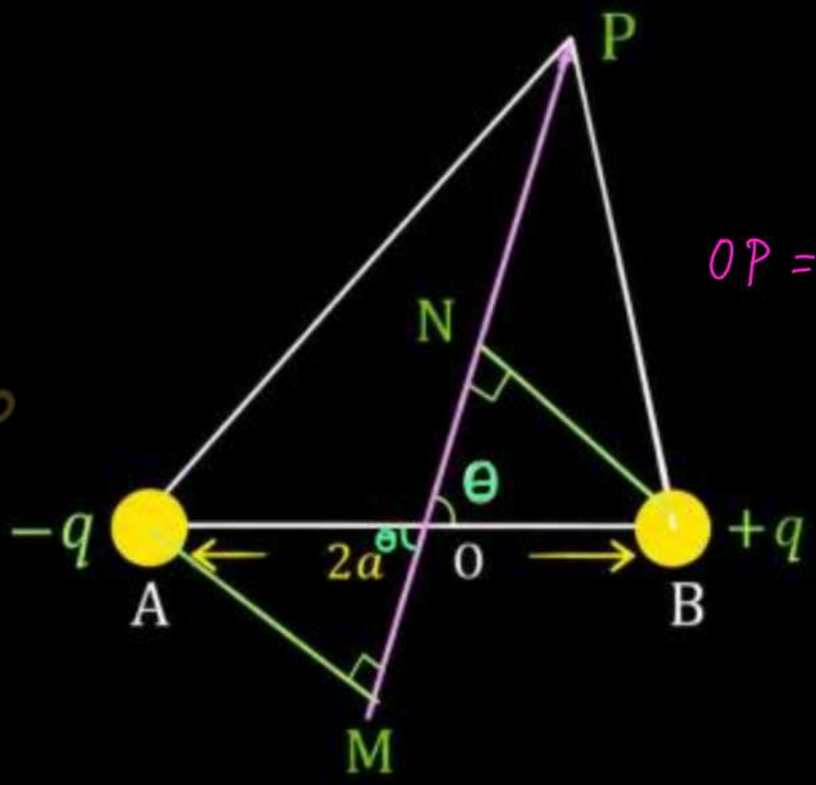
Potential at P due to +q at A

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{AP}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{MP}\right)$$

Resultant Potential

$$V = V_1 + V_2$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{NP} + \frac{(-q)}{MP}\right)$$



$OP = r$

$BP = NP$

$AP = MP$

$ON = OM = a\cos\theta$

Gateway Classes

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{NP} + \frac{(-q)}{MP}\right)$$

NOW, $\quad NP = OP - ON$

$$= r - a\cos\theta$$

$$MP = MO + OP$$

$$= a\cos\theta + r$$

$$= r + a\cos\theta$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r - a\cos\theta} - \frac{q}{r + a\cos\theta}\right)$$

$$= \frac{2aq\cos\theta}{4\pi\varepsilon_0\,(r^2 - a^2\cos^2\theta)}$$

$2aq = p \rightarrow$ dipole moment

$$\therefore \quad V = \frac{p\cos\theta}{4\pi\varepsilon_0\,(r^2 - a^2\cos^2\theta)}$$

As $r >> a$, $\therefore$ $a^2\cos^2\theta$ term may be neglected in comparison to r^2.

$\therefore$

$$\boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}}$$

Gateway Classes

Que 1) Two dipoles with dipole moments $-5\hat{a}_z$ nC.m and $9\hat{a}_z$ nC.m are located at points (0,0,-2) and (0,0,3) respectively. Find the potential at the origin.

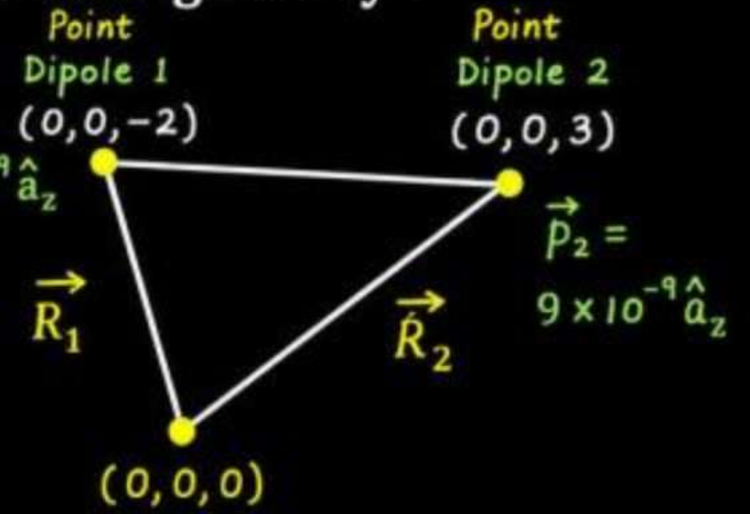
The total Electric potential V at the origin due to both the dipoles is given by :

$$V = V_1 + V_2$$

Learn

where $V_1 = \dfrac{\vec{p}_1 \cdot \hat{a}_{R_1}}{4\pi\varepsilon_0 R_1^2}$ and $V_2 = \dfrac{\vec{p}_2 \cdot \hat{a}_{R_2}}{4\pi\varepsilon_0 R_2^2}$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ \mathrm{N m^2/C^2}.$$



$\vec{R}_1 = \vec{r}_0 - \vec{r}_1 = (0-0)\,\hat{a}_x + (0-0)\,\hat{a}_y + [0-(-2)]\,\hat{a}_z = 2\,\hat{a}_z$ and $\vec{R}_2 = \vec{r}_0 - \vec{r}_2 = (0-0)\,\hat{a}_x + (0-0)\,\hat{a}_y + [0-(3)]\,\hat{a}_z = -3\hat{a}_z$

$$\hat{a}_{R_1} = \frac{\vec{R}_1}{|\vec{R}_1|} = \frac{2\,\hat{a}_z}{\sqrt{0^2+0^2+2^2}} = \hat{a}_z, \quad \text{and} \quad \hat{a}_{R_2} = \frac{\vec{R}_2}{|\vec{R}_2|} = \frac{-3\,\hat{a}_z}{\sqrt{0^2+0^2+(-3)^2}} = -\hat{a}_z$$

From Dr
$R_1 = 2$ &
$R_2 = 3$

$\vec{p}_1 = -5 \times 10^{-9}\,\hat{a}_z$ and $\vec{p}_2 = 9 \times 10^{-9}\,\hat{a}_z$

Gateway Classes

$$V_1 = \frac{\vec{P}_1 \cdot \hat{a}_{R_1}}{4\pi\varepsilon_0 R_1^2} \quad \textit{and} \quad V_2 = \frac{\vec{P}_2 \cdot \hat{a}_{R_2}}{4\pi\varepsilon_0 R_2^2}$$

$$\therefore \quad V_1 = \frac{9 \times 10^9 \times (-5 \times 10^{-9}\,\hat{a}_z) \cdot (\hat{a}_z)}{(2)^2} \quad \textit{and} \quad V_2 = \frac{9 \times 10^9 \times (9 \times 10^{-9}\,\hat{a}_z) \cdot (-\hat{a}_z)}{(3)^2}$$

$$\therefore \quad V_1 = -11.25 \text{ V} \quad \textit{and} \quad V_2 = -9 \text{ V}$$

$$\therefore \quad \text{Total potential} = V_1 + V_2$$

$$= -11.25 + (-9) \text{ V}$$

$$= -20.25 \text{ V} \quad \underline{\text{Ans.}}$$

Gateway Classes

AKTU B.Tech Sem-3

GATEWAY CLASSES

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit-2: Lec-4

Gateway Classes

Topics to be covered

- Electric Flux Lines
- Relation between $\vec{E}$ and V - Maxwell's Equation
- Energy Density in Electrostatics Field
- PYQs

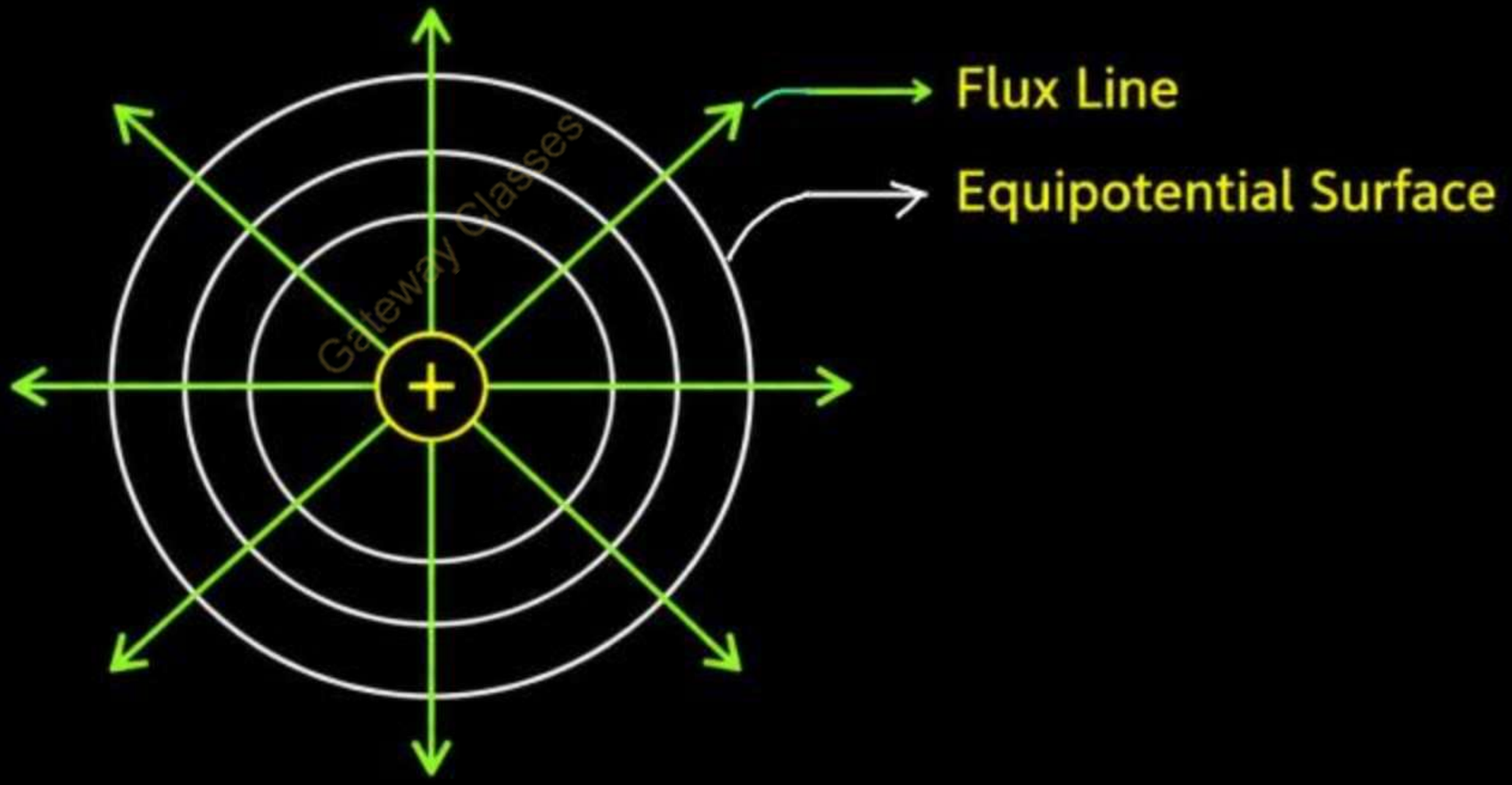
By Swati Ma'am

- M.Tech (Honours)
- B.Tech (Honours)
- GATE Qualified

Electric Flux Lines

is an imaginary path or line, drawn in such a way that its direction at any point is the direction of the electric field at that point.

Electric Flux ψ:-
Number of Electric Field lines passing through a given area.



RELATION BETWEEN $\vec{E}$ AND V – Maxwell's Equation:

$$V = -\int \vec{E} \cdot d\vec{l}$$ Imp

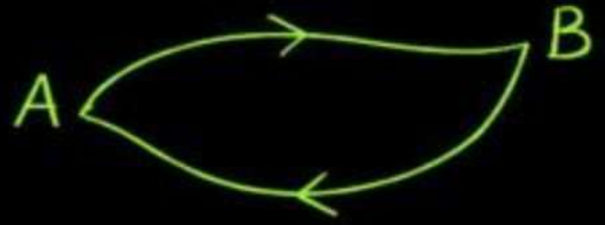
$$\boxed{\vec{E} = -\vec{\nabla} V}$$ Imp

Along a closed path ,

Maxwell's Equation in integral form states that

$$\boxed{\oint_l \vec{E} \cdot d\vec{l} = 0} \quad -(1)$$

Line integral of $\vec{E}$ along a closed path is zero.



Applying Stokes' theorem on LHS of (1)

$$\oint_l \vec{E} \cdot d\vec{l} = \int_s (\vec{\nabla} \times \vec{E}) \cdot d\vec{S} = 0$$

$$\boxed{(\vec{\nabla} \times \vec{E}) = 0}$$ → Maxwell's Eq^n

GatewayClasses

Energy density stored in Electrostatic Fields

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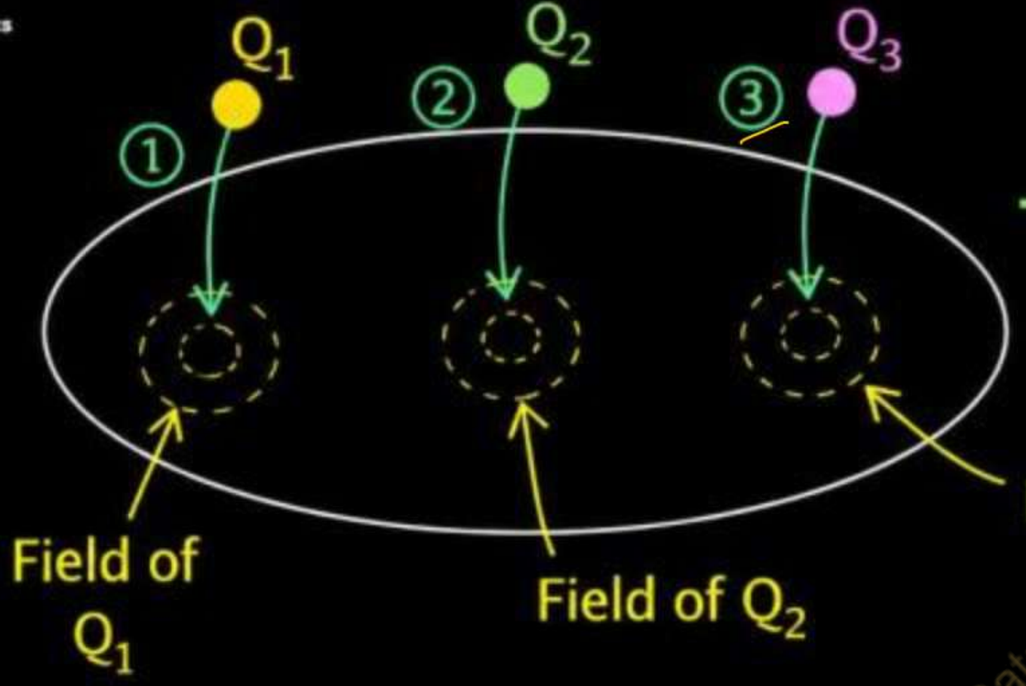


→ A surface without charge initially

Work done is represented by W or U.

GatewayClasses

① In bringing the charge Q_1 from ∞ to position 1, no work is done $W_1 = 0$

$\therefore$ field does not exist in the system

② Q_2 is brought from ∞ to position 2, $W_2 = Q_2 V_{21}$ $\therefore$ field of Q_1 exists

③ Q_3 is brought from ∞ to position 3, $W_3 = Q_3 V_{31} + Q_3 V_{32}$ $\therefore$ field of Q_1 and Q_2 exists

GATEWAY CLASSES

Total Work done

$$W_E = 0 + Q_2 V_{21} + Q_3 V_{31} + Q_3 V_{32} \quad -①$$

Now, Let us consider the three charges are brought in the reverse order i.e Q_3 is brought first, then Q_2 then Q_1.

$$W_E = 0 + Q_2 V_{23} + Q_1 V_{12} + Q_1 V_{13} \quad -②$$

Gateway Classes

① + ② ,

$$2W_E = Q_1 V_{12} + Q_1 V_{13} + Q_2 V_{21} + Q_2 V_{23} + Q_3 V_{31} + Q_3 V_{32}$$

$$= Q_1 (V_{12} + V_{13}) + Q_2 (V_{21} + V_{23}) + Q_3 (V_{31} + V_{32})$$

GATEWAY CLASSES

Learn

$$2W_E = Q_1(V_{12} + V_{13}) + Q_2(V_{21} + V_{23}) + Q_3(V_{31} + V_{32}) \quad -③$$

$$2W_E = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$$

↘ used in Numericals

where $V_1 = V_{12} + V_{13}$

Total potential at Q_1

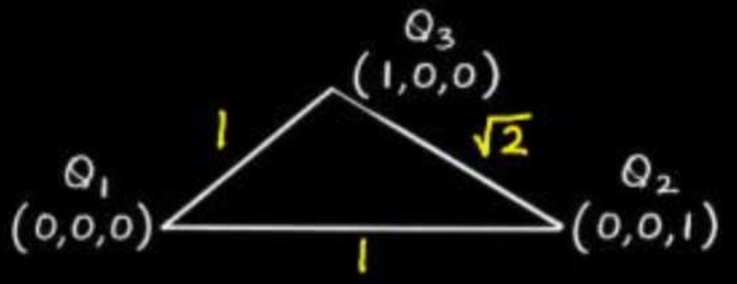
similarly V_2 & V_3

© Gateway Classes

$$\therefore \quad W_E = \frac{1}{2}\left[Q_1 V_1 + Q_2 V_2 + Q_3 V_3 \right]$$

In general, $$W_E = \frac{1}{2} \sum_{k=1}^{n} Q_k V_k$$

Que) The point charges -1 nC , 4 nC and 3 nC are located at (0,0,0), (0,0,1), and (1,0,0) respectively. Find the energy in the system.



$$W = \frac{1}{2}\sum_{k=1}^{3} Q_k V_k$$

$$= \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2 + Q_3 V_3 \right) \quad \text{where } V_1 = V_{12} + V_{13},\ V_2 = V_{21} + V_{23},\ V_3 = V_{31} + V_{32}$$

$$= \frac{Q_1}{2}\left[V_{12} + V_{13} \right] + \frac{Q_2}{2}\left[V_{21} + V_{23} \right] + \frac{Q_3}{2}\left[V_{31} + V_{32} \right] \qquad \text{From (3)}$$

Note.

$$= \frac{Q_1}{2}\left[\frac{Q_2}{4\pi\epsilon_0(1)} + \frac{Q_3}{4\pi\epsilon_0(1)} \right] + \frac{Q_2}{2}\left[\frac{Q_1}{4\pi\epsilon_0(1)} + \frac{Q_3}{4\pi\epsilon_0(\sqrt{2})} \right] + \frac{Q_3}{2}\left[\frac{Q_1}{4\pi\epsilon_0(1)} + \frac{Q_2}{4\pi\epsilon_0(\sqrt{2})} \right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[Q_1 Q_2 + Q_1 Q_3 + \frac{Q_2 Q_3}{\sqrt{2}} \right]$$

Put $Q_1 = -1 \times 10^{-9}\,C$, $Q_2 = 4 \times 10^{-9}\,C$, $Q_3 = 3 \times 10^{-9}\,C$

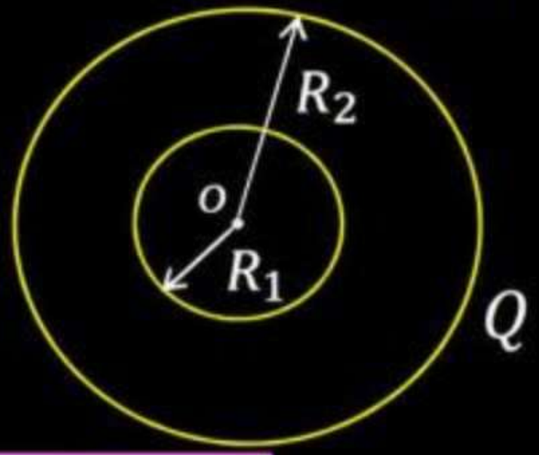
$\therefore \quad W = 13.37\ nJ$ Ans.

Gateway Classes

Energy stored in charged concentric spherical shell

R1 :- Inner Radius

R2 :- Outer Radius



Derivation :-

Learn

$$W = \frac{1}{2}\int \varepsilon_0 E^2 \, dV$$

→ differential volume

where $$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

and dV = Area × Length

Learn $dV = (4\pi r^2)\, dr$ For sphere

$$W = \frac{1}{2}\int_{R_1}^{R_2} \varepsilon_0 \frac{Q^2}{16\pi^2\varepsilon_0^2 r^4} 4\pi r^2 \, dr$$

Note

$$= \frac{Q^2}{8\pi\varepsilon_0}\int_{R_1}^{R_2} \frac{dr}{r^2}$$

$$= \frac{Q^2}{8\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{R_1}^{R_2}$$

$$W = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Gateway Classes

Energy stored in a capacitor

is the amount of work done in charging it

$$U = W_E = \int dW = \int V\,dq = \int \left(\frac{q}{C}\right) dq \qquad \therefore\ q = CV$$

$$= \frac{1}{C} \int q\ dq$$

$$= \frac{1}{C} \cdot \frac{q^2}{2} = \frac{1}{2C}(CV)^2 = \frac{1}{2} CV^2 \quad \text{—(1)}$$

Gateway Classes

Now, For a Parallel plate capacitor,

$$C = \frac{A\varepsilon_0}{d} \quad \text{and} \quad V = Ed$$

Put in (1)

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$$U = \frac{1}{2}\left(\frac{A\varepsilon_0}{d}\right)(E^2 d^2)$$

$$\boxed{U = \frac{1}{2}\varepsilon_0 E^2 A d} \text{ joules} \quad -(I)$$

Gateway Classes

Now, Ad is volume,

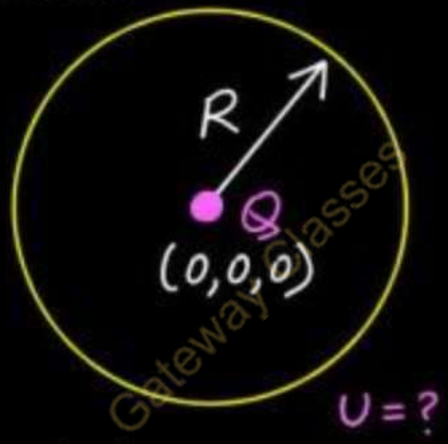
$\therefore$ Energy stored per unit volume $u = \frac{U}{Ad}$

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2} \text{ joules}/m^3$$

Que-2) A point charge Q is placed at the origin. Find the electrostatic energy stored outside the sphere of radius R centered at the origin.

The Electrostatic energy stored in the electric field is

$$U = \frac{1}{2}\epsilon_0 \int E^2 \, dV \quad -①$$



Gateway Classes

For a point charge Q at the origin, the Electric field at a distance r is

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \quad -②$$

To find energy outside a sphere of radius R

ie. from $r = R$ to $r = \infty$.

Put value of E from ② in ①

$$U = \frac{1}{2}\epsilon_0 \int_R^\infty \left(\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}\right)^2 dV$$

GATEWAY CLASSES

$$U = \frac{1}{2}\,\varepsilon_0 \int_R^{\infty} \left(\frac{1}{4\pi\varepsilon_0}\,\frac{Q}{r^2}\right)^2 dV$$

Differential volume in spherical coordinates

Put value of dV i.e $4\pi r^2 \cdot dr$

$$U = \frac{1}{2}\,\varepsilon_0 \int_R^{\infty} \left(\frac{1}{4\pi\varepsilon_0}\,\frac{Q}{r^2}\right)^2 4\pi r^2 \cdot dr$$

$$U = \frac{1}{2}\,\varepsilon_0 \cdot \frac{Q^2}{16\pi^2\varepsilon_0^2} \cdot 4\pi \int_R^{\infty} \frac{1}{r^2}\,dr$$

$$U = \frac{Q^2}{8\pi\varepsilon_0} \int_R^{\infty} \frac{1}{r^2}\,dr$$

$$= \frac{Q^2}{8\pi\varepsilon_0} \left[-\frac{1}{r}\right]_R^{\infty}$$

$$= \frac{-Q^2}{8\pi\varepsilon_0} \left[\left(\frac{1}{\infty}\right) - \left(\frac{1}{R}\right)\right]$$

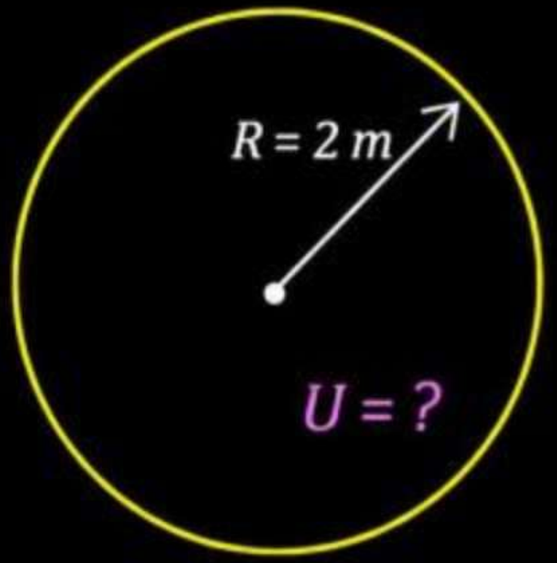
$\frac{1}{\infty} = 0$

$$= \frac{Q^2}{8\pi\varepsilon_0 R}$$

Ans

Gateway Classes

Que-3) A metallic sphere of radius 2 m is charged in air to a potential of 3000 V. Calculate the energy stored in the sphere.



$V = 3000$ volts

$$U = \frac{1}{2} C V^2$$

Learn

where capacitance C of isolated sphere is given by,

$$C = 4\pi\varepsilon_0 R$$

Learn

$$\therefore \quad U = \frac{1}{2} (4\pi\varepsilon_0 R)\ V^2$$

$$= 2\pi\varepsilon_0 R\ V^2$$

Gateway Classes

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GATEWAY CLASSES

GW
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$$U = 2 \times 3.14 \times \left(8.854 \times 10^{-12}\right) \times 2 \times (3000)^2$$

$$= 1 \times 10^{-3}\ J$$

$$= 1\ mJ$$

Ans

Gateway Classes

Que-4) For the given vector $D = x^2 y\,\hat{a}_x + \hat{a}_z$; Examine the volume charge density at (1, 1, 1) and electrostatic energy for the region $-1 < x < 1$; $-1 < y < 1$; $-1 < z < 1$.

(*i*) **Volume charge density ρ_v**

$$\overrightarrow{D} = x^2 y\,\hat{a}_x + \hat{a}_z$$

$$\rho_v = \overrightarrow{\nabla} \cdot \overrightarrow{D}$$ Learn

ρ_v ↓ scalar.

$$= \frac{\partial}{\partial x}(x^2 y) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(1)$$

$$= y(2x) + 0 + 0$$

$$\rho_v = 2xy$$

At point $(1, 1, 1)$

$$\rho_v = 2(1)(1)$$

$$= 2 \text{ C} / m^3$$

Ans

Gateway Classes

(ii) Electrostatic Energy

Energy density is

$$w = \frac{1}{2}\,\vec{D}\,.\,\vec{E}$$ Learn

where $\vec{D} = \varepsilon_0 \vec{E}$

$$\therefore \quad \vec{E} = \frac{\vec{D}}{\varepsilon_0}$$

Here $\vec{D}$ is given

$$\therefore \quad w = \frac{1}{2}\,\frac{\vec{D}\,.\,\vec{D}}{\varepsilon_0}$$ ($\vec{E}$)

$$w = \frac{1}{2\varepsilon_0} D^2 \quad -(1)$$

$$\therefore \quad \vec{D} = x^2 y\, \hat{a}_x + \hat{a}_z$$

$$D^2 = \vec{D}\cdot\vec{D} = (x^2 y \hat{a}_x + \hat{a}_z)(x^2 y \hat{a}_x + \hat{a}_z)$$

$$\therefore \quad D^2 = (x^2 y)^2 + (1)^2$$

$$D^2 = x^4 y^2 + 1$$

$\hat{a}_x \cdot \hat{a}_x = 1$
$\hat{a}_z \cdot \hat{a}_z = 1$
$\hat{a}_x \cdot \hat{a}_z = 0$
$\hat{a}_z \cdot \hat{a}_x = 0$

Gateway Classes

Energy density w ⟶ Energy → W
Volume → dV

$$w = \frac{1}{2\varepsilon_0}\left(x^4 y^2 + 1\right)$$

Energy $\boxed{W = \iiint w \; dV}$ Learn

$$W = \iiint \frac{1}{2\varepsilon_0}\left(x^4 y^2 + 1\right) dV$$

Now,

$$dV = dx\, dy\, dz$$

$W = \iiint w \cdot dV$, $dV = dx\, dy\, dz$

Gateway Classes

$$W = \iiint \frac{1}{2\varepsilon_0}\left(x^4 y^2 + 1\right) dV$$

$$= \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \frac{1}{2\varepsilon_0}\left(x^4 y^2 + 1\right) dx\, dy\, dz$$

$$= \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \frac{1}{2\varepsilon_0}\left(x^4 y^2\right) dx\, dy\, dz + \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \frac{1}{2\varepsilon_0}(1)\, dx\, dy\, dz$$

$$= \frac{1}{2\varepsilon_0}\int_{-1}^{1} x^4\, dx \int_{-1}^{1} y^2\, dy \int_{-1}^{1} dz + \frac{1}{2\varepsilon_0}\int_{-1}^{1} dx \int_{-1}^{1} dy \int_{-1}^{1} dz$$

$$= \frac{1}{2\varepsilon_0}\left[\frac{x^5}{5}\right]_{-1}^{1}\left[\frac{y^3}{3}\right]_{-1}^{1}[z]_{-1}^{1} + \frac{1}{2\varepsilon_0}[x]_{-1}^{1}[y]_{-1}^{1}[z]_{-1}^{1}$$

GW GATEWAY CLASSES
Gateway Classes

$$= \frac{1}{2\varepsilon_0} \frac{[x^5]_{-1}^{1}}{5} \frac{[y^3]_{-1}^{1}}{3} [z]_{-1}^{1} + \frac{1}{2\varepsilon_0} [x]_{-1}^{1} [y]_{-1}^{1} [z]_{-1}^{1}$$

$$= \frac{1}{2\varepsilon_0} \frac{[1-(-1)]}{5} \frac{[1-(-1)]}{3} [1-(-1)] + \frac{1}{2\varepsilon_0} (2)(2)(2)$$

$$= \frac{1}{2\varepsilon_0} \left(\frac{2}{5}\right) \left(\frac{2}{3}\right) (2) + \frac{1}{2\varepsilon_0} (8)$$

$$= \frac{4}{15\varepsilon_0} + \frac{8}{2\varepsilon_0} = \frac{8+120}{30\varepsilon_0} = \frac{128}{30\varepsilon_0} = \frac{64}{15\varepsilon_0}$$

Gateway Classes

$$W = \frac{64}{15\,\varepsilon_0}$$

$$= \frac{64}{15 \times 8.854 \times 10^{-12}}$$

$$= 4.82 \times 10^{11}\ J$$

Ans

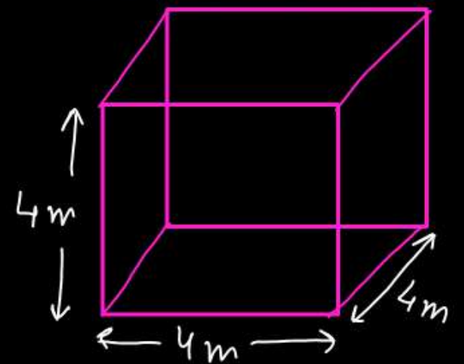
GW GATEWAY CLASSES
Gateway Classes

Que-5) If $V = yx^2 + zx + xy$ volts , do analysis of E at (2 , 3 , 7) and the electrostatic energy stored in a cube of side 4m centered at origin.

(i) $\vec{E}$

$$V = x^2 y + xy + xz$$

$$\vec{E} = -\vec{\nabla} V$$



$l = 4\,m$
$w = 4\,m$
$h = 4\,m$

given for limits of integration

$$= -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right] - (1)$$

Now, $\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left[x^2 y + xy + xz\right] = y(2x) + y(1) + z(1)$

$$= 2xy + y + z$$

© Gateway Classes

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}\left[x^2 y + xy + xz\right] = x^2(1) + x(1) + 0$$

$$= x^2 + x$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}\left[x^2 y + xy + xz\right] = 0 + 0 + x(1) = x$$

Put in (1)

$$\vec{E} = -\left[(2xy + y + z)\,\hat{a}_x + (x^2 + x)\,\hat{a}_y + (x)\,\hat{a}_z\right] \text{ -(2)}$$

Gateway Classes

At $(2, 3, 7)$

$$\vec{E} = -\left[(2)(2)(3) + 3 + 7\right]\hat{a}_x + \left[(2)^2 + 2\right]\hat{a}_y + \left[2\right]\hat{a}_z$$

$$= -22\,\hat{a}_x - 6\,\hat{a}_y - 2\,\hat{a}_z \quad \text{V/m} \quad \textit{Ans}$$

(ii) U or W

$$U = \frac{1}{2}\varepsilon_0 \int E^2\, dV$$

∴ cube is given
∴ $dV = dx\,dy\,dz$

$$U = \frac{1}{2}\varepsilon_0 \iiint E^2\, dV$$

$$= \frac{1}{2}\varepsilon_0 \iiint (2xy + y + z)^2 + (x^2 + x)^2 + (x)^2\, dV \quad \text{(From ②)}$$

4 4 4 → cube of side 4 m is given

$$= \frac{1}{2}\varepsilon_0 \int_0^4\int_0^4\int_0^4 (2xy + y + z)^2 + (x^2 + x)^2 + (x)^2\, dx\, dy\, dz$$

0 0 0

Now,

$$\int_0^4\int_0^4\int_0^4 (2xy + y + z)^2\, dx\, dy\, dz = \frac{1792}{3}$$

$$\int_0^4\int_0^4\int_0^4 (x^2 + x)^2\, dx\, dy\, dz = \frac{2816}{15}$$

Gateway Classes

$$\int_0^4 \int_0^4 \int_0^4 (x)^2 \cdot dx\, dy\, dz = \frac{128}{3}$$

$$U = \frac{1}{2} \varepsilon_0 \left[\frac{1792}{3} + \frac{2816}{15} + \frac{128}{3} \right]$$

$$= \frac{6208}{15} \varepsilon_0$$

$$= \frac{6208}{15} \left(8.854 \times 10^{-12} \right)$$

$$= 3.66 \times 10^{-9} \; J \quad \text{Ans}$$

Gateway Classes

AKTU B.Tech Sem-3

GATEWAY CLASSES

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit-2: Lec-5

Gateway classes

Topics to be covered

- **Electric Field in Material Space :-**
  - Properties of materials
  - Conductors
  - Polarization in dielectrics
  - Dielectric Constants
- **PYQs**

By Swati Ma'am

- M.Tech (Honours)
- B.Tech (Honours)
- GATE Qualified

Properties of Materials

Electric fields can exist in

- **free space**
- **material media**

Material media are of two types: Conductors, Dielectrics

1) Conductors (Metal)

$\sigma \gg 1$

(conductivity)

$$\sigma \propto \frac{1}{\text{temp}}$$

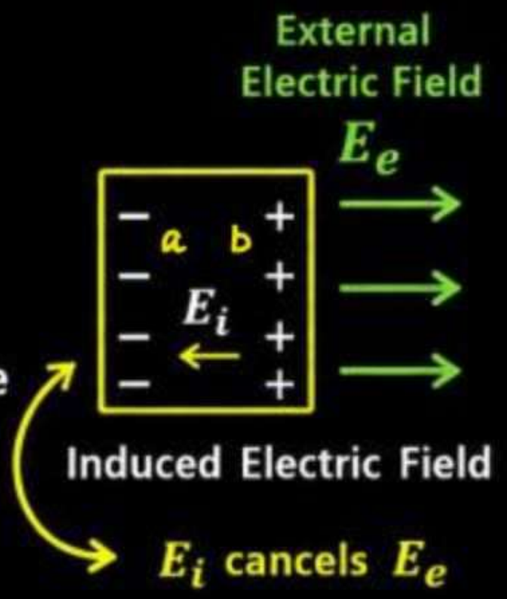
Superconductors:

At temperature near absolute zero ($T = 0\ K$), some conductors exhibit **infinite conductivity.**

eg: **Lead, Al**

Gateway Classes

CONDUCTORS



A perfect conductor $(\sigma = \infty)$ cannot contain an electrostatic field within it.

A conductor is called an equipotential body , means that the potential is the same everywhere inside it.

$$\vec{E} = -\vec{\nabla} V$$
$$= 0$$

Since V = Constant

$$\therefore \vec{E} = 0, \qquad \rho_v = 0, \qquad V_{ab} = 0$$ inside a conductor

points a and b in conductor

Gateway classes

Que 1) What is conductance ?

AKTU PYQ-2022-23

Conductance is a measure of how easily an electric current flows through a material. It is the exact opposite of electrical resistance—a material that allows electrons to pass easily has high conductance and low resistance.

Conductance (denoted by the symbol **G**) is mathematically defined as the reciprocal of resistance (R):

$$G = \frac{1}{R}$$

Using Ohm's Law, it can also be expressed as the ratio of electrical current (I) to voltage (V):

$$G = \frac{I}{V}$$

Gateway classes

Unit: **Siemens (S)**. One Siemens equals one ampere per volt.

Historical/Alternate Unit: Mho (ohm spelled backwards, Ω^{-1})

AKTU PYQ-2019-20

Que-2) Explain behaviour of a conductor at high frequency.

At low frequencies or Direct Current (DC), current flows evenly across the entire cross-section of a conductor.

At high frequencies, an electrical conductor's behavior changes dramatically due to the skin effect, where current is forced to flow only along the outer surface of the wire.

At high frequencies, the alternating magnetic field generated by the current induces "eddy currents" that oppose the current flow at the center of the wire.

This decreases the usable cross-sectional area, drastically increases the AC resistance, and causes signals to encounter significant inductive and capacitive parasitics.

Gateway Classes

2) Dielectrics

- No. of **free electrons** is low

  Examples: *Mica, Glass, Plastic*
- When placed in an **electric field**, a dielectric acts as a source of **storage of electrical charges.**
- To be a good dielectric, the material must be an **insulator.** → $\sigma \ll 1$

All dielectrics are insulators but not all insulators are dielectrics.

(i) Dielectric Constant ε_r (Relative Permittivity)

ε_r → epsilon

When a dielectric is placed between the plates of a capacitor, its **capacity** (to store charge) is increased.

$$\varepsilon_r = \frac{C}{C_0} = \frac{V_0}{V_d}$$

Where:

- C: Capacitance of the capacitor **with dielectric**
- C_0: Capacitance of the capacitor **without dielectric**
- V_0: Potential difference (between plates of capacitor) **without dielectric**
- V_d: Potential difference (between plates of capacitor) **with dielectric**

Gateway Classes

Relation using Coulomb's Law

We know:

$$F_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

Now, when charges are placed in some other medium:

$$F = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2}$$

Where:

$\varepsilon =$ **permittivity** of medium (dielectric)

Thus,

$$\frac{F}{F_0} = \frac{\varepsilon_0}{\varepsilon} = \frac{1}{\varepsilon_r}$$

Therefore,

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{F_0}{F}$$

$\epsilon = \epsilon_0 \epsilon_r$

Note:

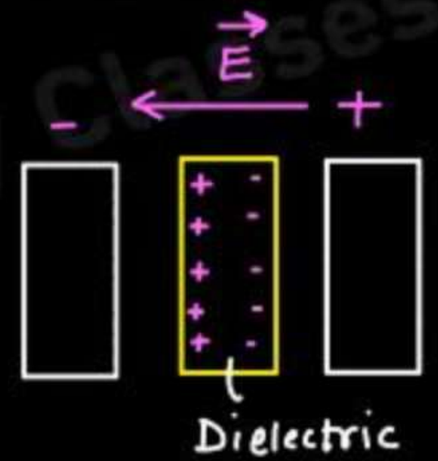
Permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F/m}$

Gateway Classes

$\epsilon_r = 1$ for **vacuum**

$\epsilon_r = \infty$ for **metals**

(ii) Electric Susceptibility χ → chi



When a dielectric is placed in electric field, it is **polarised**.

The capability of dielectric to withstand the electric field before breakdown is called χ.

The **polarisation vector** $\vec{P}$ is proportional to the electric field $\vec{E}$. Hence,

$$\vec{P} \propto \vec{E}$$

$$\vec{P} = \chi \vec{E}$$

$$|\vec{P}| = (\epsilon_0(\epsilon_r - 1))E$$

χ

where

χ → Electric susceptibility

↳ Ratio of **polarisation vector** to the **electric field intensity** in the dielectric

Also,

$$\chi = \epsilon_0(\epsilon_r - 1) = \frac{\vec{P}}{\vec{E}}$$

Note :-

$$\chi = \epsilon_0 \epsilon_r - \epsilon_0$$

$$\frac{\chi + \epsilon_0}{\epsilon_0} = \epsilon_r$$

$$1 + \frac{\chi}{\epsilon_0} = \epsilon_r$$

$\frac{\chi}{\epsilon_0} \rightarrow \chi_e$

Gateway Classes

Que 3)

Given:

$$\chi = 35.4 \times 10^{-12}$$

Find ε_r and ε.

We know:

$$\boxed{\chi = \varepsilon_0(\varepsilon_r - 1)}$$

So,

$$\varepsilon_r - 1 = \frac{\chi}{\varepsilon_0}$$

$$\varepsilon_r = 1 + \frac{\chi}{\varepsilon_0}$$

Substitute:

$$\varepsilon_r = 1 + \frac{35.4 \times 10^{-12}}{8.85 \times 10^{-12}}$$

$$\varepsilon_r = 1 + 4 = 5$$

Now,

$$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$$

$$\varepsilon = (8.85 \times 10^{-12}) \times 5$$

$$\varepsilon = 44.3 \times 10^{-12}\,\mathrm{C/N{\cdot}m^2}$$

Gateway Classes

Polarization in Dielectrics

(Insulating material → very poor conductor → electric current ≈ zero)

Charges in a dielectric are **not able to move** about freely.

↓

But when an **external force (field)** is applied, displacement occurs in atoms of dielectric, which consists of a **negative charge** $-q$ and a **positive charge** $+q$.

↓

When an **electric field** $\vec{E}$ is applied, the **positive charge** is displaced from its equilibrium position in the **direction of** $\vec{E}$ by the force

↓

$$\vec{F}_+ = q\vec{E}$$

while the **negative charge** is displaced in the **opposite direction**

$$\vec{F}_- = -q\vec{E}$$

↓

A **dipole** results from the displacement of the charges, and the dielectric is said to be **polarized**.

Gateway Classes

Dipole moment $\vec{p}$

$$\vec{p} = q\vec{d}$$

where $\boldsymbol{d}$ is the distance vector from $-\boldsymbol{q}$ to $+\boldsymbol{q}$.

If there are $\boldsymbol{N}$ **dipoles** in a volume $\Delta \boldsymbol{V}$ of the dielectric, the total dipole moment due to the electric field is:

$$\vec{p} = q_1\vec{d}_1 + q_2\vec{d}_2 + \cdots + q_N\vec{d}_N = \sum_{k=1}^{N} q_k\vec{d}_k$$

Polarization ($\vec{P}$)

A measure of intensity of polarization in **coulombs per metre square**

$$\vec{P} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{N} q_k\vec{d}_k}{\Delta V}$$

or,

$$\vec{P} = \lim_{\Delta V \to 0} \frac{\vec{p}}{\Delta V}$$

→ relates $\vec{P}$ and $\vec{p}$

Gateway classes

Relation between D, E, and P

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad -①$$

For some dielectrics, $\vec{P}$ is proportional to the **applied electric field** $\vec{E}$,

$$\vec{P} = \chi_e \varepsilon_0 \vec{E} \quad -②$$

Note

where

- χ_e is the **electric susceptibility** of the material
  - ↳ is a measure of how **susceptible** (or **sensitive**) a given dielectric is to **electric fields**

Gateway Classes

Dielectric Constant and Strength

Put ② in ①:

$$\vec{D} = \varepsilon_0 \vec{E} + \chi_e \varepsilon_0 \vec{E}$$
$$= \varepsilon_0 (1 + \chi_e) \vec{E}$$
$$= \varepsilon_0 \varepsilon_r \vec{E}$$

ε_r Note

where

$$\varepsilon_r = 1 + \chi_e$$

and

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

Thus,

$$\boxed{\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0}}$$ Imp

1 + (susceptibility) χ_e

Dielectric Constant / Relative Permittivity

It is the ratio of the permittivity of the dielectric to that of free space.

is the maximum electric field that a dielectric can tolerate or withstand without electrical breakdown.

Gateway Classes

AKTU B.Tech Sem-3

GATEWAY CLASSES

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit-2: Lec-6

Gateway classes

Topics to be covered

- Convection and Conduction Current
- Continuity Equation
- Relaxation time
- PYQs

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Current :-

Current is defined as the flow of charge.

$$I = \frac{q}{t}$$

Scalar Quantity

Unit : Ampere (A)

When current varies with time , then at any instant t ,

$$I = \frac{dq}{dt}$$

Gateway classes

Current Density :- $\vec{J}$

It is represented by $\vec{J}$ and is defined as the ratio of current to the cross-sectional area.

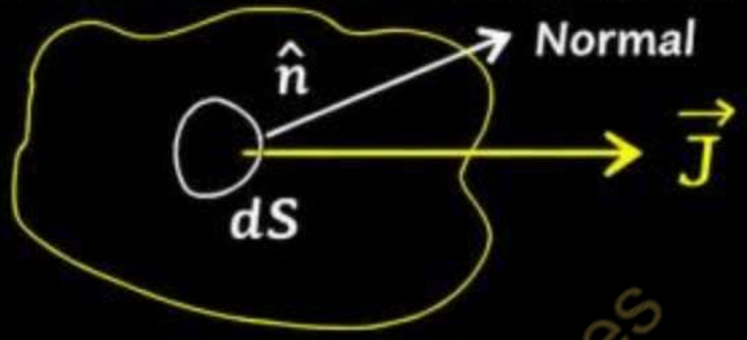
$$|\vec{J}| = \frac{I}{A}$$

It is a vector quantity whose direction at any point is the direction of positive charge movement.

Unit : Ampere per square metre (A/m^2)

Differential Form

Consider a small surface element dS with a normal unit vector $\hat{n}$



$$dq = \vec{J} \cdot \hat{n}\, dS$$

$\hat{n}\, dS \rightarrow \vec{dS}$

Total charge per unit time

$$\frac{q_t}{t} = \oint \vec{J} \cdot \vec{dS}$$

Therefore total current

$$I = \oint_s \vec{J} \cdot \vec{dS}$$

Gateway classes

Relation with Volume Charge Density

$$\vec{J} = \rho_v V$$

where $\vec{J}$ = Current Density

ρ = Volume charge density

V = Volume

v.imp

Continuity Equation

It relates Current density $\vec{J}$ and Current I.

Integral Form

$$I = \int_s \vec{J} \cdot d\vec{S} \quad -(1)$$

Point (Differential) Form

$$\vec{\nabla} \cdot \vec{J} = -\frac{d\rho_v}{dt}$$

Derivation

v.imp

Conservation of charge (1) Divergence Theorem are used.

According to the Law of Conservation of Charge, the outward flow of positive charge must be balanced by a decrease in positive charge within a closed surface.

If q is the charge inside a closed surface , then the rate of decrease of charge is :

$$\text{Rate of charge decrement} = -\frac{dq}{dt} = I$$ ↘ outward current

Gateway classes

And charge inside a volume $q = \iiint \rho_v \, dv$

Taking time derivative $I = \oint_s \vec{J} \cdot d\vec{S} = -\frac{dq}{dt} = -\frac{d}{dt} \iiint \rho_v \, dv$

From Integral form of continuity Eq^n

$$\oint_s \vec{J} \cdot d\vec{S} = -\frac{d}{dt} \iiint \rho_v \, dv = \iiint \left(-\frac{d\rho_v}{dt}\right) dv$$

Gateway classes

Now applying Gauss Divergence Theorem to the left side

$$\iiint (\vec{\nabla} \cdot \vec{J}) \, dv = \iiint \left(-\frac{d\rho_v}{dt}\right) dv$$

Since the volumes are identical

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{d\rho_v}{dt}}$$

Differential form of continuity Eq^n.

Conduction Current

AKTU PYQ-2025-26,2021-22

Conduction Current requires a conductor.

It satisfies Ohm's Law

Conduction Current Density

$$\vec{J} = \sigma \vec{E}$$

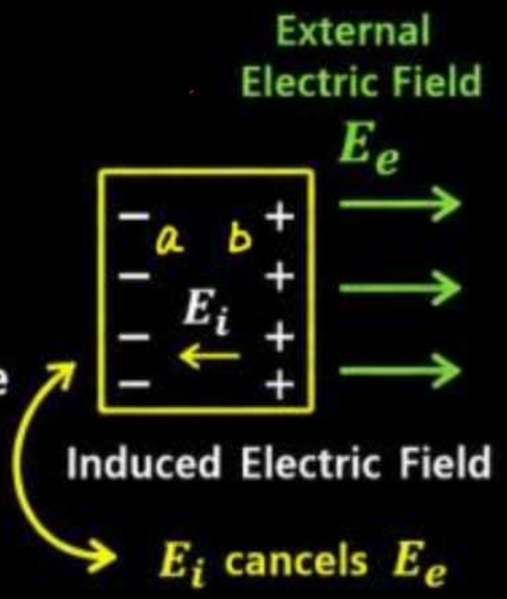
Gateway classes

where

σ = conductivity of conductor

E = Electric field

CONDUCTORS



A perfect conductor $(\sigma = \infty)$ cannot contain an electrostatic field within it.

A conductor is called an equipotential body , means that the potential is the same everywhere inside it.

$$\vec{E} = -\vec{\nabla} V$$
$$= 0$$

Since V = Constant

$$\therefore \vec{E} = 0, \qquad \rho_v = 0, \qquad V_{ab} = 0$$

inside a conductor

$V_{ab} = 0$ → points a and b in conductor

Gateway classes

Convection Current

AKTU PYQ-2025-26,2021-22

Convection current does not involve conductors

It occurs when current flows through an insulating medium such as :

Liquid

Rarefied gas

Vacuum

Example :-

A beam of electrons in a vacuum tube is an example of convection current.

It does not satisfy Ohm's Law.

Gateway classes

Convection Current Density

$$\vec{J} = \rho_v \vec{u}$$

$\vec{u}$: charge velocity

where

ρ_v = volume charge density

$\vec{u}$ = Charge velocity

Displacement Current

AKTU PYQ-2022-23

It is a **conceptual current** arising from a **time-varying Electric Field.**

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

where Φ_E is Electric Flux.

The total current is the sum of conduction current i_c and displacement current i_d.

i.e., $I = i_c + i_d$.

In terms of conduction current density J_c and displacement current density J_d, i_c and i_d can be expressed as

$$i_c = \int_S J_c \cdot dS \quad \text{and} \quad i_d = \int_S J_d \cdot dS$$

Gateway classes

(iii) $I = J \times A$

where $A = \pi r^2$ of Wire

$\therefore I = 5 \times 10^{5} \times 3.14 \times (0.5 \times 10^{-3})^2$

$= 0.3925 \ A$ Ans

Gateway classes

Relaxation Time τ

AKTU PYQ-2020-21,2019-20

Relaxation time τ is the time required for an unbalanced charge density inside a material to decay to e^{-1} (about 36.8%) of its initial value.

Also called Re-arrangement time.

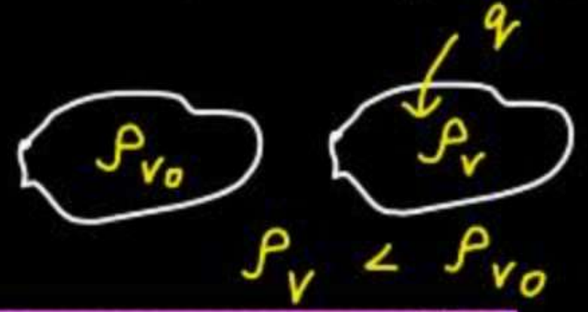
$$\left(\because \frac{1}{e} = \frac{1}{2.718} = 0.368 \right)$$

For good conductors , this time is extremely short , while for insulators ,it is very long.

$$\tau = \frac{\varepsilon}{\sigma}$$

where ε is permittivity

σ is conductivity



$$\rho_v = \rho_{v_0} e^{-t/\tau}$$

Gateway classes

Que-1) A wire of diameter 1 mm and conductivity 5×10^7 S/m has 10^{29} free electrons per cubic meter when an electric field of 10 mV/m is applied. Find :-

(i) Charge density of free electrons

(ii) Current density

(iii) Current in the wire

Diameter $(d) = 1\ mm = 1 \times 10^{-3}\ m$

$\therefore$ Radius $(r) = \frac{d}{2} = 0.5 \times 10^{-3}\ m$

$\sigma = 5 \times 10^7\ S/m$

$n = 10^{29}$ per m^3

$E = 10\ mV/m = 10 \times 10^{-3}\ V/m$

$e = 1.6 \times 10^{-19}\ C$

(i) $\rho_v = n \times e$

$= 10^{29} \times 1.6 \times 10^{-19}$

$= 1.6 \times 10^{10}\ C/m^3$ Ans

(ii) $J = \sigma E$

$= (5 \times 10^7)(10 \times 10^{-3})$

$= 5 \times 10^5\ A/m^2$ Ans

Gateway classes

Que-2) Given the current density , $J = \frac{1}{r^2}(2\cos\theta\,\hat{a}_r + \sin\theta\,a_\theta)$ A/m^2 ,find current through a spherical shell of r = 10 cm.

$$I = \iint \vec{J} \cdot d\vec{S}$$

where $\vec{dS} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$ in spherical co-ordinate system

$$\therefore I = \iint \frac{1}{r^2}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta)(r^2\sin\theta\, d\theta\, d\phi\, \hat{a}_r)$$

$$= \iint (2\cos\theta\sin\theta\, d\theta\, d\phi\, \hat{a}_r\hat{a}_r) + \iint (\sin\theta \sin\theta\, d\theta\, d\phi\, \hat{a}_\theta\hat{a}_r)$$

$2\cos\theta\sin\theta = \sin 2\theta$; $\hat{a}_r\hat{a}_r = 1$; $\hat{a}_\theta\hat{a}_r = 0$

Gatewayclasses

$$I = \iint_{\pi}^{2\pi} \sin 2\theta \, d\theta \, d\phi$$

$$= \int_{\theta=0}^{\pi} \sin 2\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \left[-\frac{\cos 2\theta}{2} \right]_0^{\pi} \left[\phi \right]_0^{2\pi}$$

$$= -\frac{1}{2} \left[\cos 2\theta \right]_0^{\pi} \left[\phi \right]_0^{2\pi}$$

$$= -\frac{1}{2} \left[\cos 2\pi - \cos 0 \right] \left[2\pi - 0 \right]$$

($\cos 2\pi \to 1$, $\cos 0 \to 1$)

$$= -\frac{1}{2} \left[1 - 1 \right] \left[2\pi \right]$$

$$= 0$$ Ans

GW GATEWAY CLASSES
Gateway classes

AKTU B.Tech Sem-3

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GATEWAY CLASSES

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit-2: Lec-7

Gateway classes

Topics to be covered

- Capacitance
- imp Boundary Conditions
- PYQs

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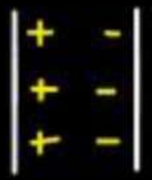
Capacitance

Capacitor:

↳ device or arrangement which can store **considerable amount of charge ,** also called **Condenser**

It consists of two  conductors next to each other & carrying equal & opposite charge.

Capacitance

is the ratio of magnitude of charge q on either conductor to the potential V between them.

$$C = \frac{q}{V}$$

$$\frac{1 \text{ coulomb}}{1 \text{ volt}} = 1 \text{ Farad}$$

Farad is **very large**. ∴ for practical purposes μF (micro-farad) is used.

Gateway Classes

Combination (of n Capacitors)

- **Series combination**

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

- **In parallel combination**

$$C = C_1 + C_2 + C_3 + \cdots + C_n$$

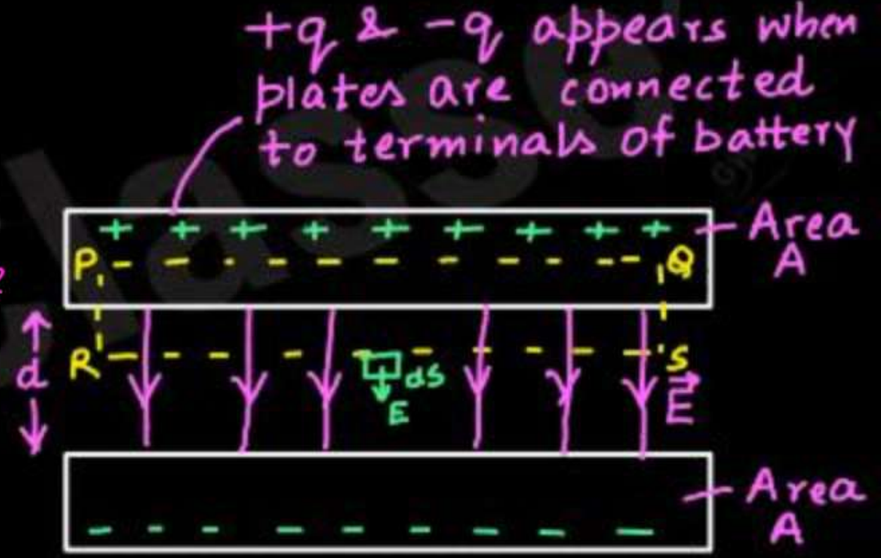
Gateway Classes
Gateway classes

1) Capacitance of a Parallel Plate Capacitor

The plates are connected to terminals of a battery.

Formula

$$C = \frac{\epsilon_0 A}{d}$$



Steps To find C:

1. Consider a Gaussian surface
2. Apply Gauss law & find E (Electric Field)
3. Find V (Potential) & put the value of E
4. Find C (Capacitance) & put the value of V

Gaussian Surface → Gauss Law → E → V → C

$$E \rightarrow V \rightarrow C$$

Gaussian surface: PQRS (closed cylinder of height h and cross-sectional area A) such that dS is ⊥ to PR and QS.

- Flux through PR = 0 ($\vec{E} \perp d\vec{S}$), Flux through QS = 0 ($\because \vec{E} \perp d\vec{S}$)
- Flux through PQ = 0 ($\vec{E}$ inside conductor = 0
- **Contribution to the flux is only due to RS.**

$\vec{dS}$ is area of PR & QS

$\phi = \oint \vec{E} \cdot \vec{dS}$

Gateway Classes

Electric Flux (Φ_E):

$$\Phi_E = \oint \vec{E} \cdot d\vec{S} = \oint E dS \cos\theta$$

Φ_E ↙ due to RS

$\theta \to 0°$ as dS is parallel to RS

$$\Phi_E = \vec{E} \oint d\vec{S}$$

($\because E$ is constant)

$$\Phi_E = EA$$

Apply Gauss Law:

$$\Phi_E = \frac{q}{\epsilon_0}$$

$$\therefore \quad EA = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{\epsilon_0 A} \quad \ldots \quad (1)$$

Gateway Classes
Gateway classes

Finding Potential Difference (V)

The **potential difference** (V) between the plates is the **work done per unit charge** in moving a charge from the **negative plate** to the **positive plate**. Since the **electric field** E is **uniform** between the plates, the potential difference is simply the **product** of the **electric field** and the **separation distance** (d):

$$V = Ed$$ Note (in PPC)

Substitute the expression for E from **equation (1)**:

$$V = \left(\frac{q}{\epsilon_0 A}\right) d$$

$$V = \frac{qd}{\epsilon_0 A} \quad \dots \quad (2)$$

Gateway classes

Finding Capacitance (C)

Capacitance (C) is defined as the ratio of the magnitude of the **charge** (q) on either conductor to the **potential difference** (V) between them:

$$C = \frac{q}{V}$$

Substitute the expression for V from equation (2) into the definition of C:

$$C = \frac{q}{\left(\frac{qd}{\varepsilon_0 A}\right)}$$

$$C = q \times \frac{\varepsilon_0 A}{qd}$$

The **charge** q cancels out, leaving the final expression for the **capacitance** of a parallel plate capacitor:

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

Gateway Classes

GW GATEWAY CLASSES

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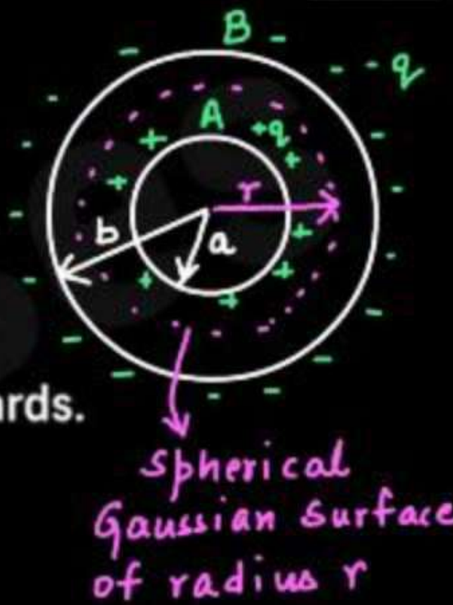
2) Capacitance of a Spherical Capacitor

The derivation is for a spherical capacitor consisting of two concentric conducting spheres.

- **Inner metal sphere** A of radius a, charge of q.
- **Outer metal sphere** B of radius b, charge of $-q$.



The charges produce an electric field E in the space between them, which is **radially outwards.**

1. Finding Electric Field (E) using Gauss's Law

- **Consider a Gaussian surface** (a sphere) of radius r $(a < r < b)$ through it.
- **Flux (Φ_E):**

$$\Phi_E = \oint \vec{E} \cdot \vec{dS} = \oint E dS \cos\theta$$

... as dS is parallel to E, $\theta = 0^\circ$, $\cos 0^\circ = 1$

$$\Phi_E = \vec{E} \oint \vec{dS} = E(4\pi r^2) \quad \text{(Area of Gaussian surface)}$$

spherical Gaussian surface of radius r

Gateway Classes

- **Apply Gauss Law:**

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

ϕ_E by Gauss Law

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

2. Finding Potential Difference (V)

We know, $V = -\int E dr$ $(\because E = -\frac{dV}{dr})$, i.e., the potential difference between two spheres is equal to the **work done** in transferring a unit positive charge from outer sphere to inner sphere thus,

Note

$$V = V_A - V_B = -\int_b^a E \cdot dr \quad \begin{pmatrix} a \to \text{inner radius} \\ b \to \text{outer radius} \end{pmatrix}$$

Note:- Limits $b \to a$

$$\therefore V = -\int_b^a \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr$$

Gateway Classes

$$= -\frac{q}{4\pi\epsilon_0}\int_b^a \frac{dr}{r^2}$$

$$= -\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_b^a$$

$$= -\frac{q}{4\pi\epsilon_0}\left(-\frac{1}{a}+\frac{1}{b}\right) = +\frac{q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{b-a}{ab}\right]$$

$$C = \frac{q}{V}$$

imp

$$\boxed{C = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)} \quad \text{or} \quad C = 4\pi\epsilon_0\left(\frac{1}{\frac{1}{a}-\frac{1}{b}}\right)$$

Capacitance of Isolated Conducting Sphere

When outer surface is of infinite radius i.e., $b = \infty$

$$C = \frac{4\pi\epsilon_0}{\left(\frac{1}{a}-\frac{1}{\infty}\right)}$$

$$\boxed{C = (4\pi\epsilon_0)a}$$

GW GATEWAY CLASSES
Gateway classes

3) Capacitance of Cylindrical Capacitor

Inner radius : a

Outer radius : b

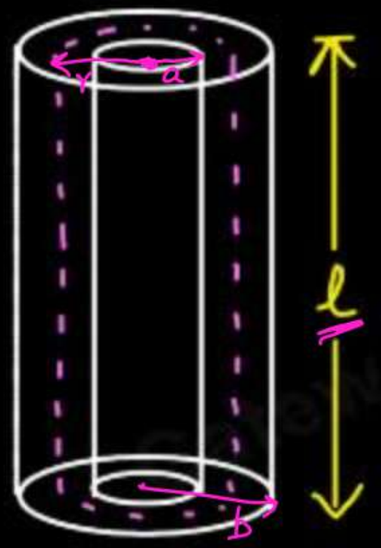
Consider cylindrical Gaussian surface of radius r.

Apply **Gauss law**:

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

$$E(2\pi r l) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{2\pi\epsilon_0 r l} \quad \ldots(1)$$



$\int d\vec{S} = 2\pi r l$
↳ area of Gaussian surface

Potential difference

$$V = -\int_b^a \vec{E}\, dr$$

Note:- Limits b → a

$$V = -\frac{q}{2\pi\epsilon_0 l}\int_b^a \frac{dr}{r}$$

$$V = -\frac{q}{2\pi\epsilon_0 l}[\log r]_b^a$$

$$V = -\frac{q}{2\pi\epsilon_0 l}(\log a - \log b)$$

$$V = \frac{q}{2\pi\epsilon_0 l}\log\left(\frac{b}{a}\right)$$

Capacitance.

$$C = \frac{q}{V}$$

$$\boxed{C = \frac{2\pi\epsilon_0 l}{\log(b/a)}}$$

Gateway Classes

Que 1) The **flux density** within the **cylindrical volume** bounded by

$$\rho = 2\,\text{m}, \quad z = 0 \text{ and } z = 5\,\text{m}$$ → limits

is given by

$$\vec{D} = 30e^{-\rho}\,\hat{a}_\rho - 2z\,\hat{a}_z$$

What is the total **outward flux** crossing the **surface of** the **cylinder**?

Solution

According to Gauss's law: → when $\vec{D}$ is given, $\Phi = ?$

$$\Psi = \iint_S \vec{D} \cdot d\vec{S}$$

For a **cylindrical surface** (side surface):

$$d\vec{S} = \rho\, d\phi\, dz\, \hat{a}_\rho$$ in Cylindrical Co-ordinates

Thus,

$$\Psi = \iint_S (30e^{-\rho}\,\hat{a}_\rho - 2z\,\hat{a}_z) \cdot (\rho\, d\phi\, dz\, \hat{a}_\rho)$$

Gateway Classes

Since

$$\hat{a}_\rho \cdot \hat{a}_\rho = 1, \quad \hat{a}_\rho \cdot \hat{a}_z = 0$$

we get:

$$\Psi = \int_0^{2\pi} \int_0^5 30\, e^{-\rho}\, \rho\, dz\, d\phi$$

Given $\rho = 2$:

$$\Psi = \int_0^{2\pi} \int_0^5 30 \cdot 2 \cdot e^{-2}\, dz\, d\phi$$

$$= 60e^{-2} \int_0^{2\pi} d\phi \int_0^5 dz$$

$$= 60e^{-2}(2\pi)(5)$$

$$\boxed{\Psi = 600\pi e^{-2}}$$

Numerically ($e^{-2} \approx 0.1353$):

$$\Psi \approx 255.1 \quad \text{Nm}^2/\text{C} \text{ or Volt meters}$$

GW GATEWAY CLASSES
Gateway Classes

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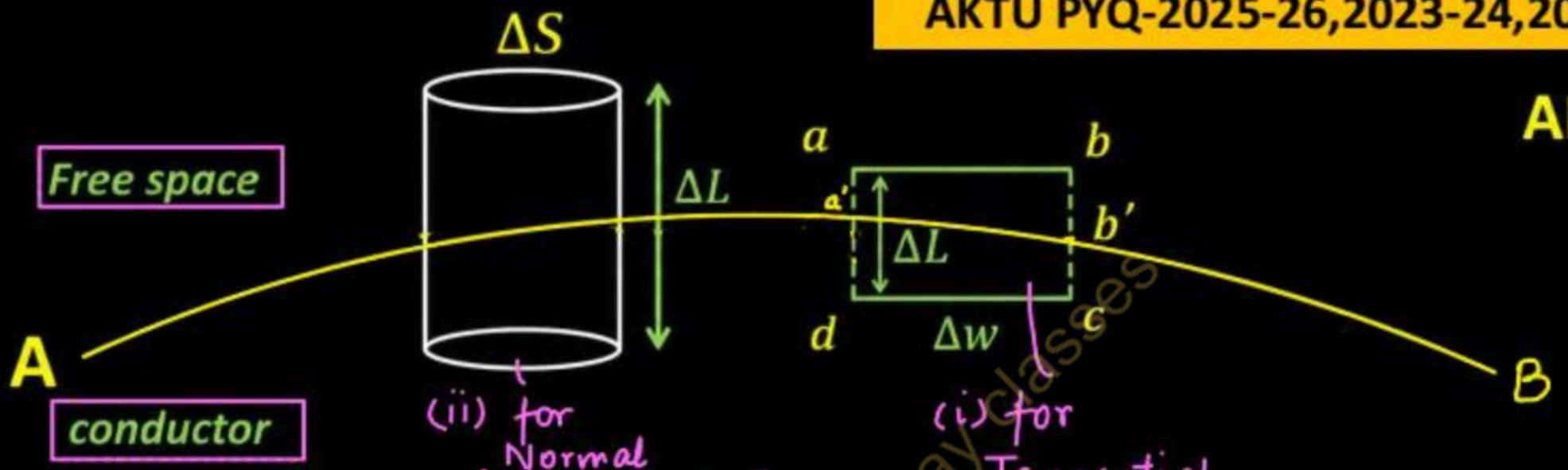
Boundary Conditions for Electric Field

$E_t = ?$
$E_N = ?$

AKTU PYQ-2025-26,2023-24,2020-21,2019-20



AB :- Boundary between conductor and free space.

start from $\oint \vec{D} \cdot d\vec{S} = q$

start $\vec{E} \cdot d\vec{l} = 0$

(i) Tangential Components

Consider a Gaussian surface $abcda$.

Let us move a unit positive charge along this closed path.

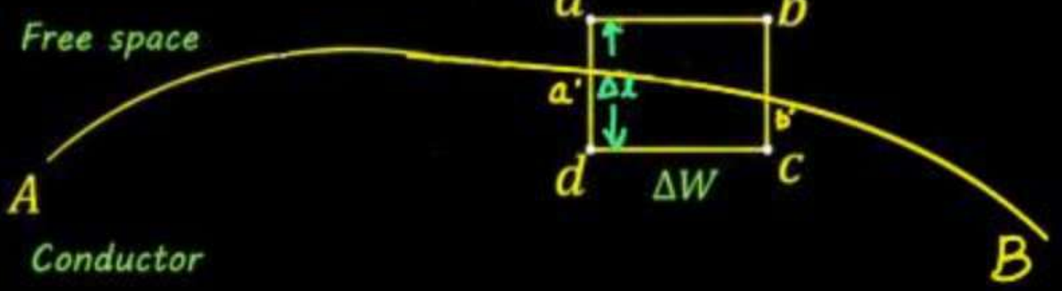
No work is required.

$\because$ displacement $= 0$ $\therefore$ $\vec{E} \cdot d\vec{l} = 0$] Note

Gateway Classes

$$\int \vec{E} \cdot d\vec{\ell} = 0$$

$$\int_a^b \vec{E} \cdot d\vec{\ell} + \int_b^c \vec{E} \cdot d\vec{\ell} + \int_c^d \vec{E} \cdot d\vec{\ell} + \int_d^a \vec{E} \cdot d\vec{\ell} = 0$$



$$\int_a^b \vec{E} \cdot d\vec{\ell} + \underbrace{\left[\int_b^{b'} \vec{E} \cdot d\vec{\ell} + \int_{b'}^{c} \vec{E} \cdot d\vec{\ell}\right]}_{0 \text{ ↳ inside conductor}} + \underbrace{\int_c^d \vec{E} \cdot d\vec{\ell}}_{0 \text{ ↳ inside conductor}} + \underbrace{\left[\int_d^{a'} \vec{E} \cdot d\vec{\ell} + \int_{a'}^{a} \vec{E} \cdot d\vec{\ell}\right]}_{0} = 0$$

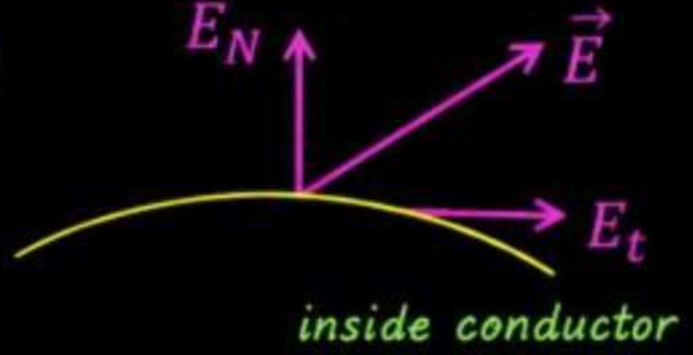
$\because$ inside conductor

$$E_t \; \Delta W + \left[-E_N \left(\frac{1}{2}\Delta\ell\right)\right] + E_N \left(\frac{1}{2}\Delta\ell\right) = 0$$

E_t : $\because a \to b$; E_N : $b \to b'$; E_N : $a' \to a$

$$E_t \; \Delta W = 0$$

$$\Delta W \neq 0 \quad \therefore \quad \boxed{E_t = 0}$$



Learn

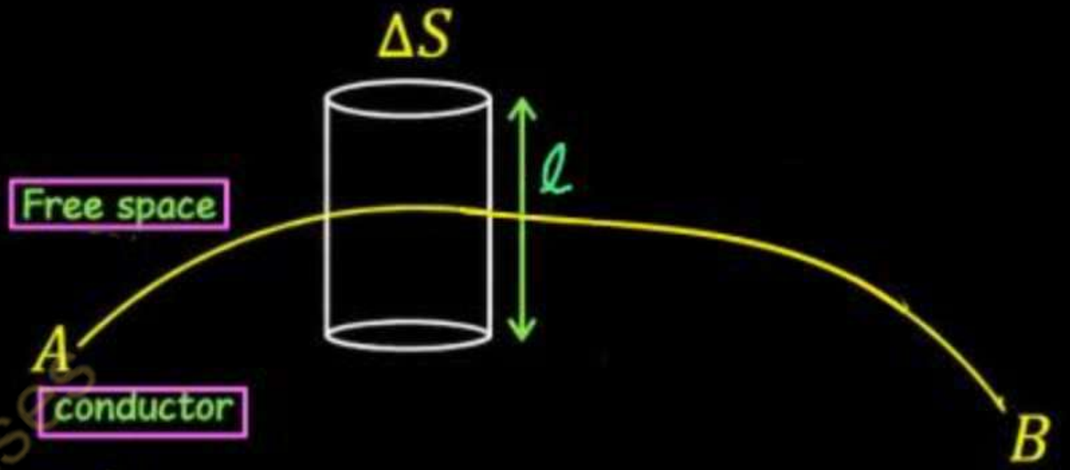
Gateway classes

(ii) Normal Components E_N

Consider a cylindrical Gaussian surface.

Applying Gauss Law,

$$\oint \vec{D} \cdot d\vec{S} = q$$ Note



$$\int_{Top} \vec{D} \cdot d\vec{S} + \underbrace{\int_{Bottom} \vec{D} \cdot d\vec{S}}_{0 \text{ inside conductor}} + \int_{\text{Side surface in Free space}} \vec{D} \cdot d\vec{S} + \underbrace{\int_{\text{Side surface in Conductor}} \vec{D} \cdot d\vec{S}}_{0} = q$$

$\therefore$ *inside conductor*

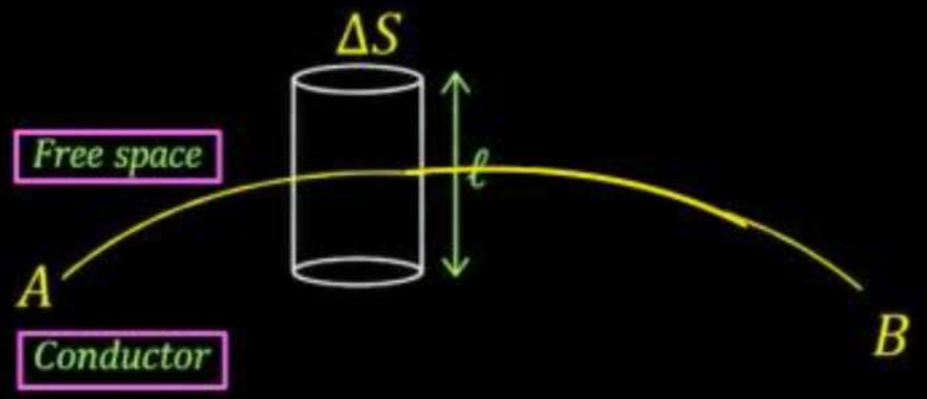
$$\therefore \int_{Top} \vec{D} \cdot d\vec{S} + \int_{\text{Side surface in Free space}} \vec{D} \cdot d\vec{S} = q$$

Gateway Classes

$$\int_{\text{Top}} \vec{D} \cdot d\vec{S} + \int_{\substack{\text{Side surface} \\ \text{in Free space}}} \vec{D} \cdot d\vec{S} = q$$

$\because$ area of side surface is negligible.

$d\vec{S} = 0$



$$\therefore \int_{\text{Top}} \vec{D} \cdot d\vec{S} = q$$

$D_N \Delta S = q$ (Top)

$= \rho_s \Delta S$ ⇢ surface charge density

$\rho_s = \frac{q}{\Delta S} \Rightarrow q = \rho_s \Delta S$

$\therefore$ $\boxed{D_N = \rho_s}$

$\because D = \epsilon_0 E$

$D_N = \epsilon_0 E_N = \rho_s$

$\therefore$ $\boxed{E_N = \frac{\rho_s}{\epsilon_0}}$ Learn

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AKTU B.Tech Sem-3

GATEWAY CLASSES

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit-2: Lec-8

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Topics to be covered

- Electrostatic boundary value problems :- Poisson's and Laplace's Equations
- Method of Images
- PYQs

By Swati Ma'am

- M.Tech (Honours)
- B.Tech (Honours)
- GATE Qualified

Poisson and Laplace Equations

AKTU PYQ-2021-22,2020-21

Methods to find Electric Field Intensity

used to find $\vec{E}$, V

(i) Gauss's Law

→ But it is impracticable when the charge distribution is not known.

(ii)

$$\vec{E} = -\nabla V$$

→ But it is impracticable when the potential throughout the region is not known.

Therefore:

Poisson and Laplace Equations

- Fundamental differential equations
- Applicable in solving those problems which cannot be solved by Gauss's law
- Used for calculating electric field and potentials subjected to boundary conditions

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Poisson's Equation

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

Laplace's Equation

$$\nabla^2 V = 0$$

RHS = 0

(i) In Cartesian Coordinate System

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

(ii) In Cylindrical Coordinate System

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

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(iii) Spherical Coordinates

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

V. imp

Derivation of Poisson's Eq^n :-

According to Gauss's Law,

$$\oint_s \vec{D} \cdot d\vec{s} = q \qquad (1)$$

$\rho_v = \frac{q}{V}$

$$= \int_V \rho_v \, dV \qquad (2)$$

Maxwell's Ist Eq^n

Keypoint
Gauss Law
Div Theorem
$\vec{D}$
Poissons
Laplace

(If the charge is distributed in the volume V over the surface of which the integral is taken)

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By Gauss divergence theorem

LHS

$$\oint_S \vec{D} \cdot d\vec{s} = \int_V \vec{\nabla} \cdot \vec{D}\, dV \quad (3)$$

From (2) & (3):

$$\int_V \vec{\nabla} \cdot \vec{D}\, dV = \int_V \rho_v\, dV$$

$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho_v} \quad (4)$$

(This is Maxwell's first eqn.)

We know,

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{D} = \epsilon(-\vec{\nabla} V)$$

$$\vec{D} = -\epsilon \vec{\nabla} V \quad (5)$$

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Substituting the value of $\vec{D}$ from (5) in (4)

$$\nabla \cdot (-\varepsilon \vec{\nabla} V) = \rho_v$$

$$-\varepsilon \nabla^2 V = \rho_v$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$ Poisson's Eqⁿ

This is known as **Poisson's equation**

↓

If there is no charge present, then $\rho_v = 0$

$$\boxed{\nabla^2 V = 0}$$ Laplace Eqⁿ

This is known as **Laplace's equation**

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Que-1) What is the physical significance of Poisson and Laplace Equation ?

Poisson and Laplace Equations

- Fundamental differential equations
- Applicable in solving those problems which cannot be solved by Gauss's law
- Used for calculating electric field and potentials subjected to boundary conditions

$\vec{E}$ V

Gateway classes

Que-2)

Find potential and volume charge density at $P(0.5, 1.5, 1)$ in free space of potential field $V = (x^2 - y^2 - z^2)$ volt.

Sol'n

(i) $V = [(0.5)^2 - (1.5)^2 - (1)^2] = -3$ volt. Ans

(ii) **Volume charge density** ρ_v. $\nabla^2 V = \frac{-\rho_v}{\epsilon_0}$ (Poisson's eqn̂)

$$\rho_v = -\epsilon_0(\nabla^2 V)$$

$$= -\epsilon_0\left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right]$$

$$= -\epsilon_0[2 - 2 - 2]$$

$$= 2\epsilon_0$$

$$= 17.708 \times 10^{-12}\ C/m^3 \text{ Ans}$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2}{\partial x^2}(x^2 - y^2 - z^2) = 2$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{\partial^2}{\partial y^2}(x^2 - y^2 - z^2) = -2$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{\partial^2}{\partial z^2}(x^2 - y^2 - z^2) = -2$$

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Que-3) State whether following potential function satisfies Laplace Equation $\nabla^2 V = 0$

$V = Cr\theta$ where C = constant.

$V = Cr\theta$ is in spherical co-ordinates

$\therefore$ Laplacian operator in spherical co-ordinates.

Learn

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot \frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} \quad -(I)$$

Ist term

$$\frac{\partial V}{\partial r} = \frac{\partial}{\partial r}(Cr\theta) = C\theta\frac{\partial}{\partial r}r = C\theta(1) = C\theta$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot \frac{\partial V}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}(r \cdot C\theta) = \frac{1}{r} \cdot C\theta(1) = \frac{C\theta}{r} \quad -①$$

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Now, $\frac{\partial V}{\partial \theta}$ (IInd term) $= \frac{\partial}{\partial \theta}(Cr\theta) = Cr\frac{\partial}{\partial \theta}(\theta) = Cr(1) = Cr$

$$\frac{\partial^2 V}{\partial \theta^2} = \frac{\partial}{\partial \theta}\left(\frac{\partial V}{\partial \theta}\right) = \frac{\partial}{\partial \theta}(Cr) = 0$$

$$\frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = \frac{1}{r^2}\cdot 0 = 0 \quad -(2)$$

Put (1) and (2) in (I)

$$\nabla^2 V = \frac{C\theta}{r} + 0 = \frac{C\theta}{r}$$

$\because \nabla^2 V \neq 0 \therefore$ function does not satisfy Laplace Equation.

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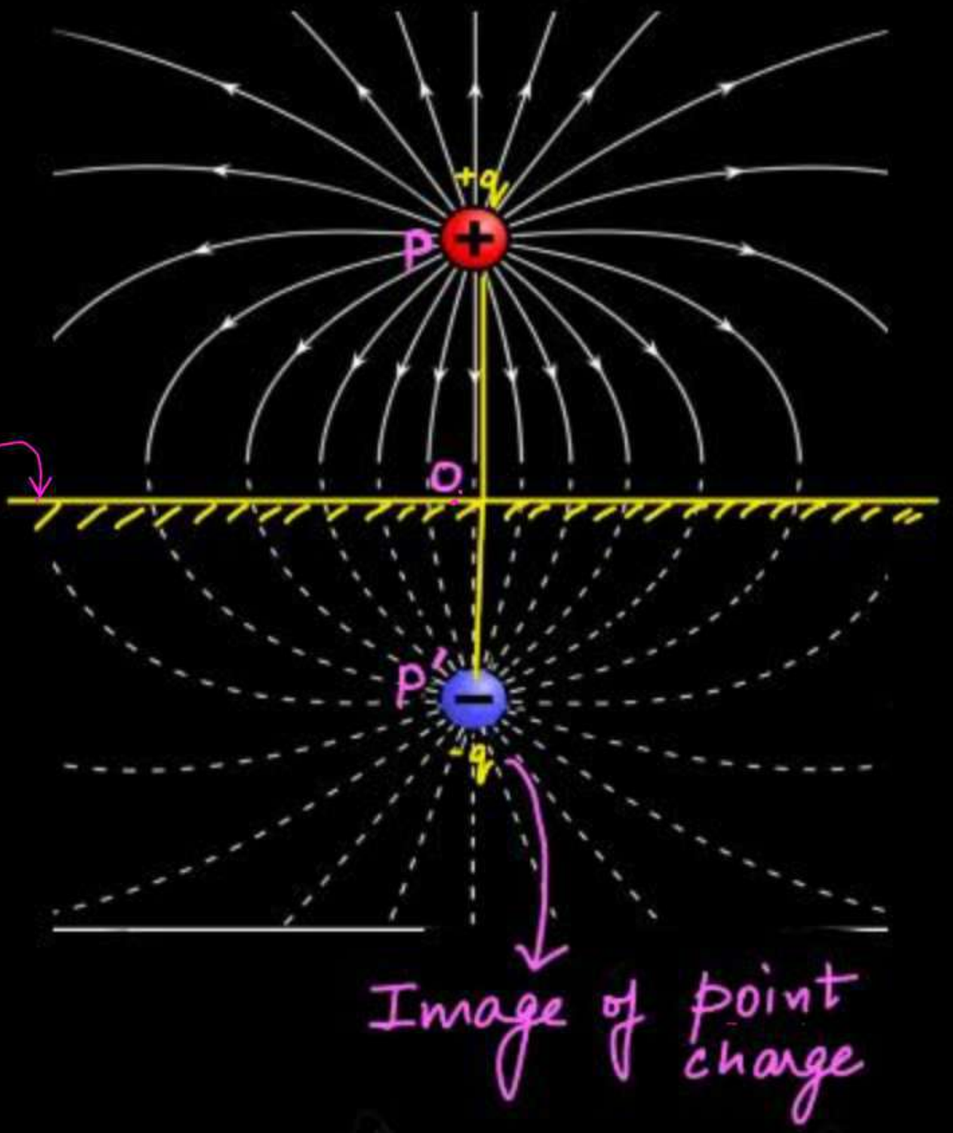
METHOD OF IMAGES

It is a simple method based on the principle of geometrical optics , used for solving electrostatic problems ,

↓

which involves point or line charges before conducting planes
or curved surfaces(cylindrical or spherical).

In this method , the conducting plane is assumed as a mirror placed instead of it.



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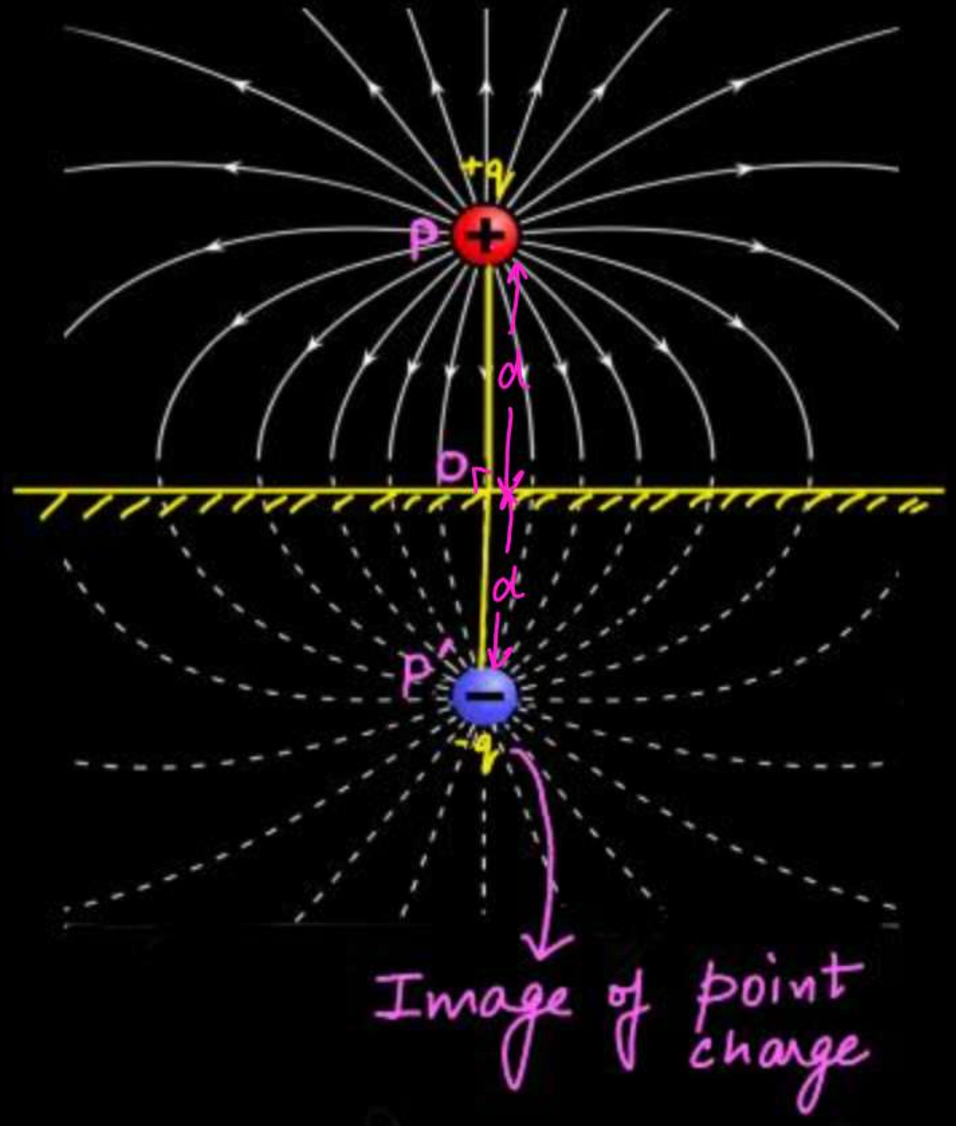
Let us consider a point charge +q placed at distance d from the conducting plane.

By method of Images,
Draw a perpendicular (OP) from point P to O on the conducting surface and extend it to the length d , so that :

$$OP' = PO = d$$

Put an image charge of opposite nature and same magnitude (i.e –q) at point P' and hypothetically remove the actual conducting plane. Now both the charges form a dipole whose electric field can be easily found.



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Thank You

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